

AP Calculus AB - Riemann Sums and the Definite Integral - Answer Key

Common Core AP CALC AB 6.1, AP CALC AB 6.2, AP CALC AB 6.3 · maths-pdf.fr/en/

This answer key gives a full worked solution for every problem in the riemann sums worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 6.1, AP CALC AB 6.2, AP CALC AB 6.3**

Exercise 1 - Left Riemann sum

$\Delta x = 1$. Left endpoints 0,1,2,3: $f = 1, 2, 5, 10$.

Sum: $1(1+2+5+10) = 18$.

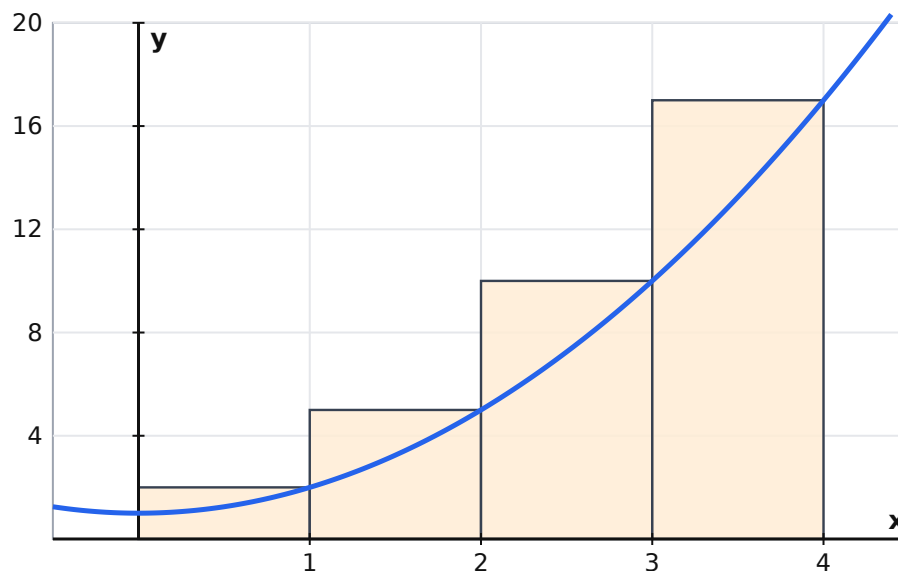
Final answer: 18.

Exercise 2 - Right Riemann sum

Right endpoints 1,2,3,4: $f = 2, 5, 10, 17$.

Sum: $1(2+5+10+17) = 34$.

Final answer: 34.



Four right rectangles reach above the curve.

Exercise 3 - Midpoint Riemann sum

Midpoints 0.5, 1.5, 2.5, 3.5: $f = 1.25, 3.25, 7.25, 13.25$.

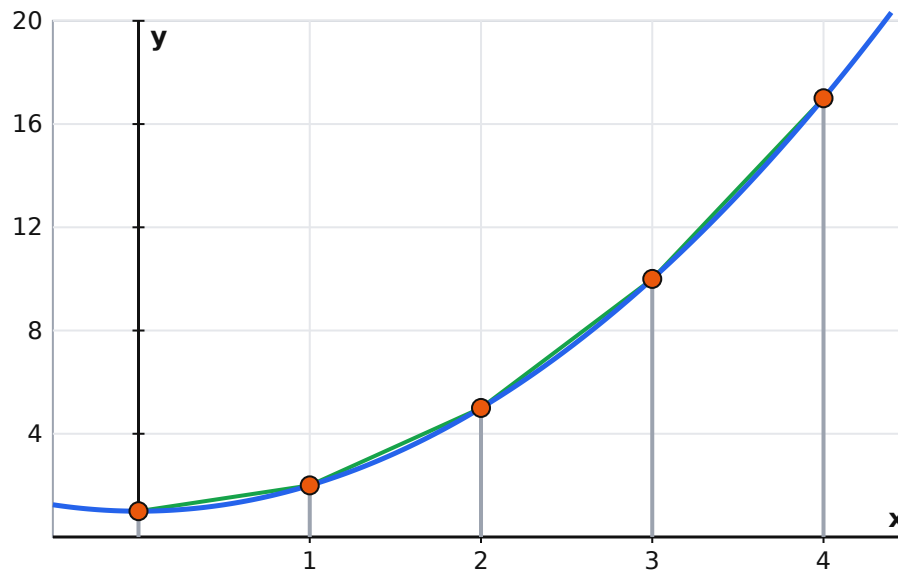
Sum: $1(1.25 + 3.25 + 7.25 + 13.25) = 25$. (The exact area is $\frac{76}{3} \approx 25.33$.)

Final answer: 25.

Exercise 4 - Trapezoidal sum

Trapezoidal sum $= \frac{\text{left} + \text{right}}{2} = \frac{18 + 34}{2} = 26$.

Final answer: 26.



Four trapezoids.

Exercise 5 - Riemann sums from a table

Each width is 2. Left: $2(3+5+8+12) = 56$. Right: $2(5+8+12+17) = 84$. Trapezoidal: $\frac{56+84}{2} = 70$.

Final answer: 56, 84, 70 feet.

Exercise 6 - Unequal subintervals

Widths: 1, 3, 2, 4. Left sum: $1(2) + 3(4) + 2(5) + 4(9) = 2 + 12 + 10 + 36 = 60$.

Trapezoidal: $\frac{2+4}{2}(1) + \frac{4+5}{2}(3) + \frac{5+9}{2}(2) + \frac{9+12}{2}(4) = 3 + 13.5 + 14 + 42 = 72.5$.

Final answer: left 60; trapezoidal 72.5.

Exercise 7 - Over or under

f is increasing, so each left rectangle lies below the curve: **underestimate**.

Left sum: $1(0+1+\sqrt{2}+\sqrt{3}) \approx 0+1+1.414+1.732 = 4.146$.

Final answer: ≈ 4.15 (underestimate).

Exercise 8 - Sigma notation

$\Delta x = \frac{1}{2}$ and the left endpoints are $x_i = \frac{i}{2}$ for $i = 0, 1, 2, 3$.

$$\sum_{i=0}^3 \left(\frac{i}{2}\right)^2 \cdot \frac{1}{2} = \frac{1}{2} \left(0 + \frac{1}{4} + 1 + \frac{9}{4}\right) = \frac{1}{2}(3.5) = 1.75.$$

Final answer: 1.75.

Exercise 9 - Limit as an integral

$\Delta x = \frac{2}{n}$ means the interval has length 2. The sample points $x_i = 1 + \frac{2i}{n}$ start at 1 and end at 3.

Final answer: $\int_1^3 x^2 dx$.

Exercise 10 - Triangle area

The region is a triangle with base 4 and height $f(4) = 8$.

$$\text{Area: } \frac{1}{2}(4)(8) = 16.$$

Final answer: 16.

Exercise 11 - Semicircle

The graph is the upper half of the circle $x^2 + y^2 = 9$ with radius 3.

$$\text{Area: } \frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}.$$

Final answer: $\frac{9\pi}{2} \approx 14.14$.

Exercise 12 - Signed area from a graph

From -2 to 2 : a triangle above the axis with base 4 and height 4: area 8.

From 2 to 4 : a triangle below the axis with base 2 and height 2: signed area -2 .

Final answer: $8 - 2 = 6$.

Exercise 13 - Properties of integrals

$$\int_0^7 f = 5 + (-2) = 3. \quad \int_7^0 f = -3. \quad \int_0^3 2f = 2(5) = 10.$$

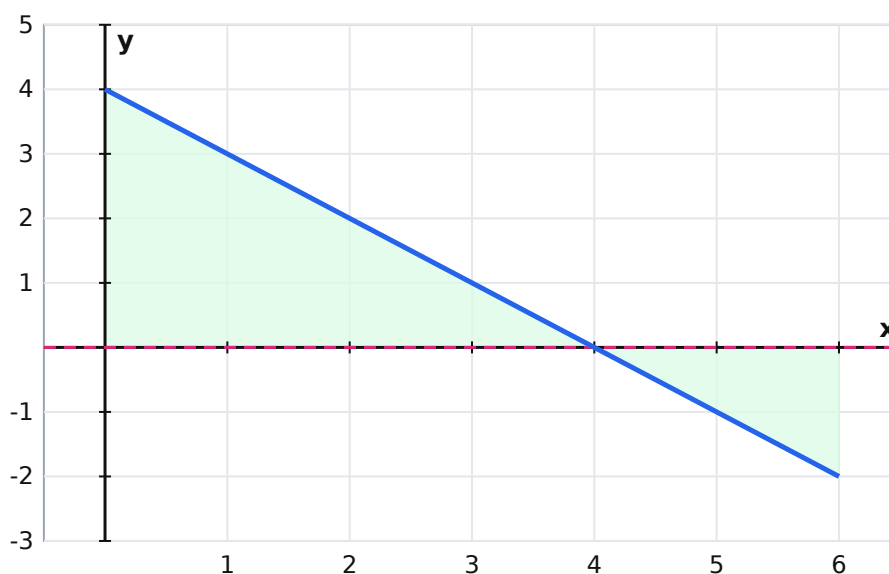
Final answer: 3, -3, 10.

Exercise 14 - Net change versus distance

From 0 to 4 the velocity is positive: triangle with base 4, height 4: area 8. From 4 to 6 it is negative: triangle base 2, height 2: signed area -2 .

Displacement: $8 - 2 = 6$. Distance: $8 + 2 = 10$.

Final answer: displacement 6 m; distance 10 m.



Areas above and below the axis.

Exercise 15 - Rainfall estimate

Width 0.25 h. Trapezoids: $0.25 \left[\frac{0+2}{2} + \frac{2+5}{2} + \frac{5+4}{2} + \frac{4+1}{2} \right] = 0.25(1+3.5+4.5+2.5) = 0.25(11.5) = 2.875$.

Final answer: ≈ 2.88 inches.

Exercise 16 - Electricity use

$$2 \left[\frac{1.2+2.0}{2} + \frac{2.0+3.4}{2} + \frac{3.4+2.6}{2} \right] = 2(1.6+2.7+3.0) = 2(7.3) = 14.6.$$

Final answer: 14.6 kWh.

Exercise 17 - Bracketing the exact value

$\Delta x = 0.5$. Left: $0.5(0+0.125+1+3.375) = 0.5(4.5) = 2.25$. Right:
 $0.5(0.125+1+3.375+8) = 0.5(12.5) = 6.25$.

$$2.25 < 4 < 6.25.$$

Final answer: left 2.25, right 6.25.

Exercise 18 - Limit of right sums

$$\Delta x = \frac{2}{n}, x_i = \frac{2i}{n}. \text{ Sum: } \sum_{i=1}^n \frac{2i}{n} \cdot \frac{2}{n} = \frac{4}{n^2} \cdot \frac{n(n+1)}{2} = \frac{2(n+1)}{n} = 2 + \frac{2}{n}.$$

As $n \rightarrow \infty$, $\frac{2}{n} \rightarrow 0$.

Final answer: 2 (the area of the triangle with base 2 and height 2).

Exercise 19 - Decreasing function

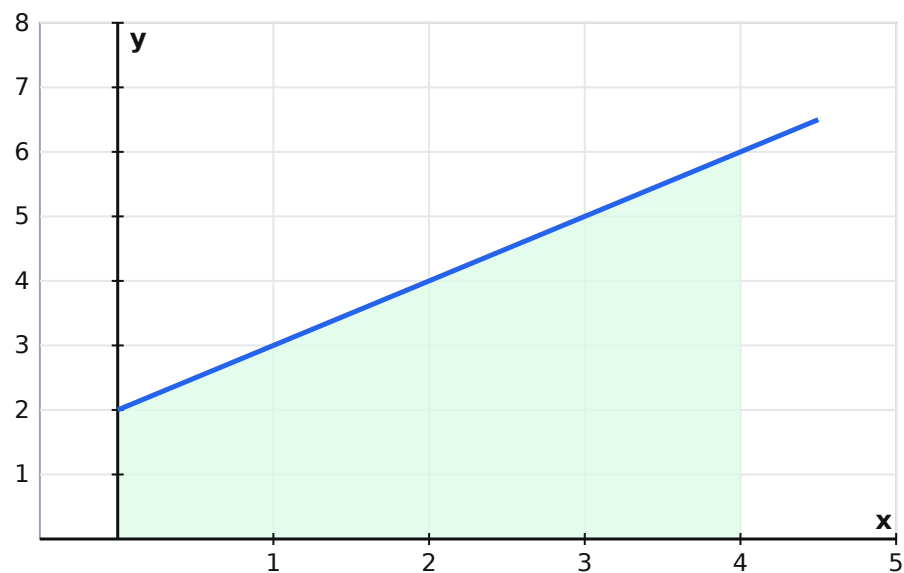
$\Delta x = 1$. Left: $f(1)+f(2)+f(3)+f(4) = 8+4+\frac{8}{3}+2 = \frac{50}{3} \approx 16.67$. Right:
 $f(2)+f(3)+f(4)+f(5) = 4+\frac{8}{3}+2+\frac{8}{5} \approx 10.27$.

For a decreasing function, the left sum overestimates and the right sum underestimates. The exact value is $8 \ln 5 \approx 12.88$.

Final answer: left ≈ 16.67 (over); right ≈ 10.27 (under).

Exercise 20 - Rectangles and trapezoids

- The region is a rectangle of width 4 and height 3: 12.
- The region is a trapezoid with parallel sides $f(0)=2$ and $f(4)=6$ and width 4: $\frac{2+6}{2} \cdot 4 = 16$.



The trapezoid under $y = x + 2$.

Exercise 21 - Highway traffic

Each width is 1 hour. $\frac{120+180}{2} + \frac{180+260}{2} + \frac{260+200}{2} + \frac{200+140}{2} = 150 + 220 + 230 + 170 = 770$.

Final answer: 770 cars.