

AP Calculus AB - Riemann Sums and the Definite Integral - Worksheet

Common Core AP CALC AB 6.1, AP CALC AB 6.2, AP CALC AB 6.3 · maths-pdf.fr/en/

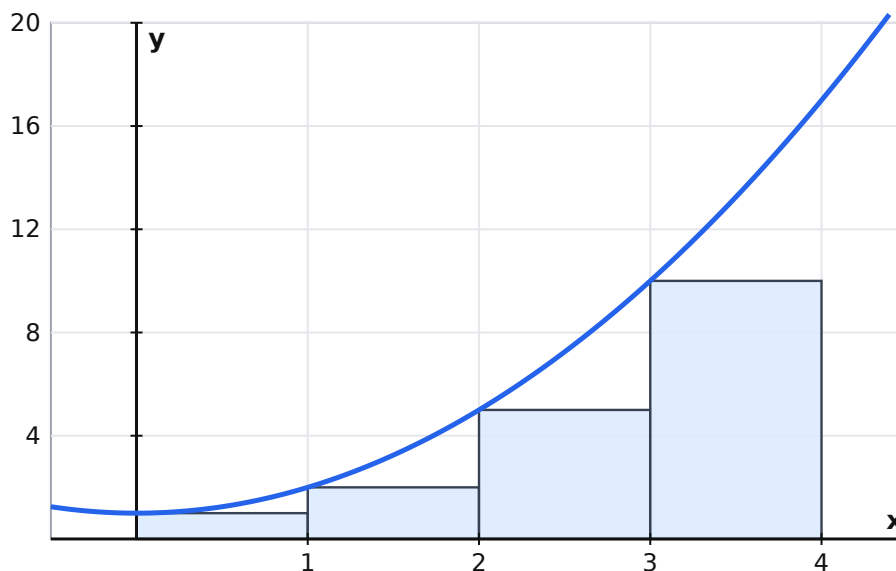
Name: _____ Class: _____ Date: ____/____/____

These 21 problems practice Riemann sums, trapezoidal sums, the definite integral as signed area and accumulation of change (AP Calculus AB, topics 6.1, 6.2, 6.3). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use tables with unequal subintervals, sigma notation, signed areas from graphs and situations such as rainfall and electricity use. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 6.1, AP CALC AB 6.2, AP CALC AB 6.3**

Exercise 1 - Left Riemann sum



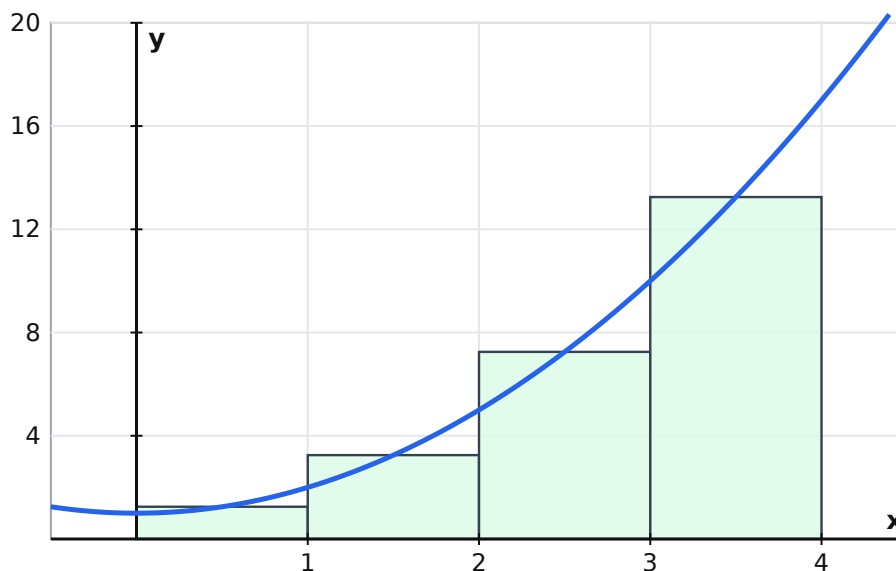
Four left rectangles for $f(x) = x^2 + 1$.

Find the left Riemann sum for $f(x) = x^2 + 1$ on $[0, 4]$ with $n = 4$.

Exercise 2 - Right Riemann sum

Find the right Riemann sum for $f(x) = x^2 + 1$ on $[0, 4]$ with $n = 4$.

Exercise 3 - Midpoint Riemann sum



Midpoint rectangles.

Find the midpoint Riemann sum for $f(x) = x^2 + 1$ on $[0, 4]$ with $n = 4$.

Exercise 4 - Trapezoidal sum

Use the two sums above to find the trapezoidal sum for $f(x) = x^2 + 1$ on $[0, 4]$ with $n = 4$.

Exercise 5 - Riemann sums from a table

t (s)	0	2	4	6	8
v (ft/s)	3	5	8	12	17

Velocity of a cart.

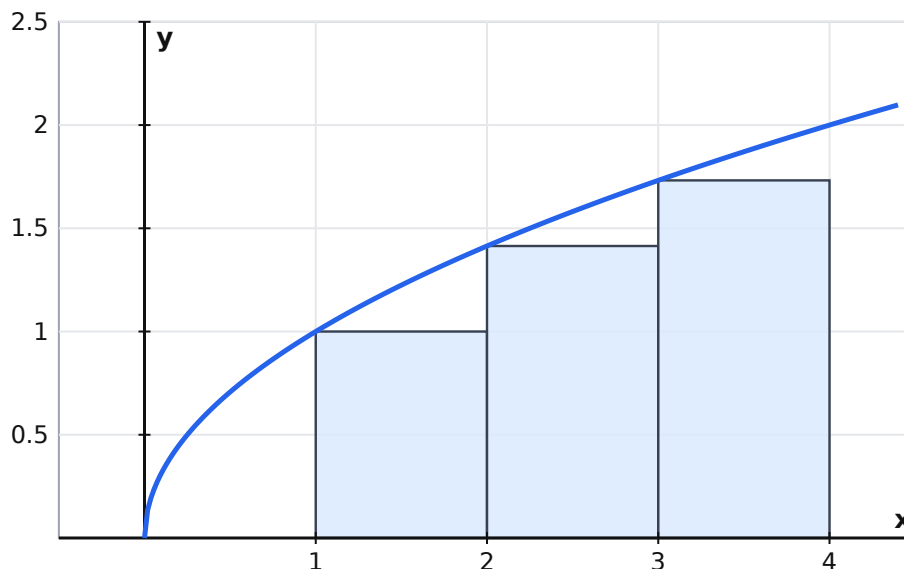
A cart's velocity is given in the table. Estimate the distance traveled from $t = 0$ to $t = 8$ with a left sum, a right sum and a trapezoidal sum using four subintervals.

Exercise 6 - Unequal subintervals

A function has $g(0) = 2$, $g(1) = 4$, $g(4) = 5$, $g(6) = 9$ and $g(10) = 12$. Estimate $\int_0^{10} g(x) dx$ with a

left sum and a trapezoidal sum.

Exercise 7 - Over or under



Left rectangles for $\sqrt{x}$.

Is the left Riemann sum for $f(x) = \sqrt{x}$ on $[0, 4]$ with $n = 4$ an overestimate or an underestimate? Compute it and compare with the true value $\frac{16}{3} \approx 5.33$.

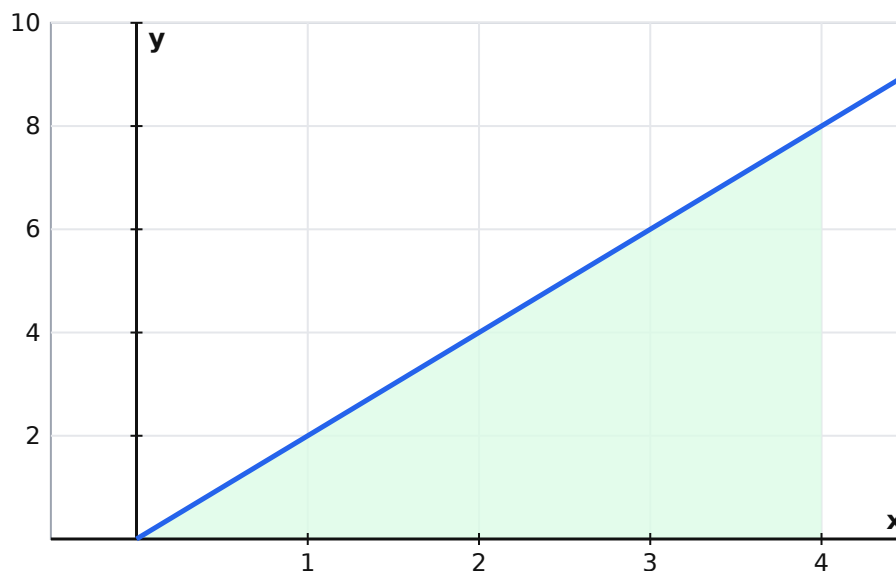
Exercise 8 - Sigma notation

Write the left Riemann sum of $f(x) = x^2$ on $[0, 2]$ with $n = 4$ in sigma notation and evaluate it.

Exercise 9 - Limit as an integral

Write $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + \frac{2i}{n}\right)^2 \frac{2}{n}$ as a definite integral.

Exercise 10 - Triangle area



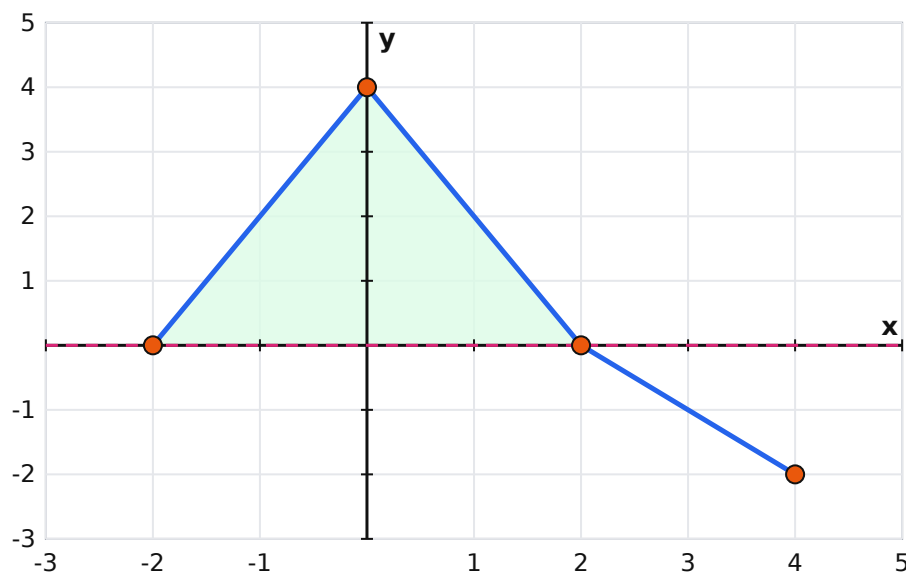
The region under $y = 2x$.

Use geometry to evaluate $\int_0^4 2x \, dx$.

Exercise 11 - Semicircle

Use geometry to evaluate $\int_{-3}^3 \sqrt{9-x^2} \, dx$.

Exercise 12 - Signed area from a graph



Piecewise linear graph of f .

The graph of f is made of line segments. Find $\int_{-2}^4 f(x) \, dx$.

Exercise 13 - Properties of integrals

Given $\int_0^3 f(x) dx = 5$ and $\int_3^7 f(x) dx = -2$, find $\int_0^7 f(x) dx$, $\int_7^0 f(x) dx$ and $\int_0^3 2f(x) dx$.

Exercise 14 - Net change versus distance

A particle has velocity $v(t) = 4 - t$ m/s for $0 \leq t \leq 6$. Find the displacement and the total distance using areas.

Exercise 15 - Rainfall estimate

t (min)	0	15	30	45	60
R (in/h)	0	2	5	4	1

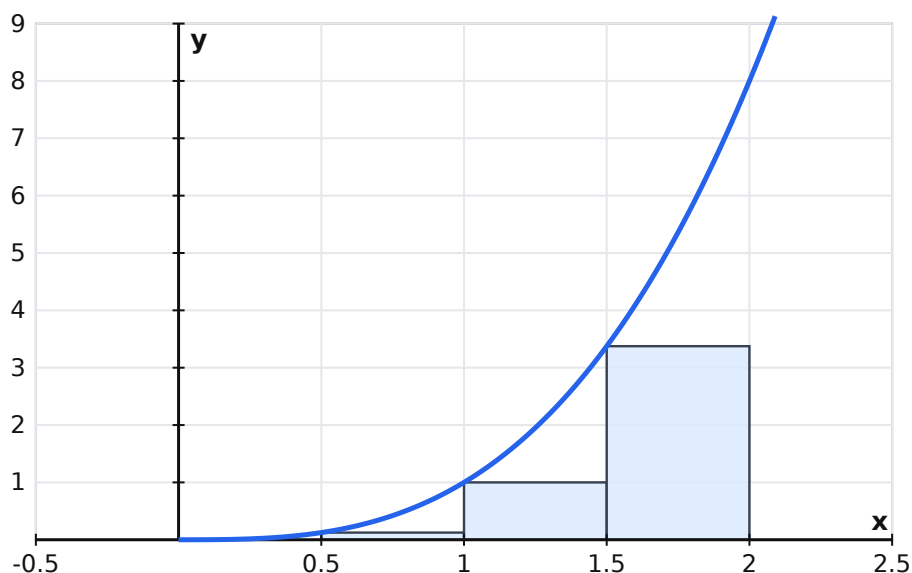
Rainfall rate measurements.

A storm's rainfall rate in inches per hour is measured every 15 minutes. Estimate the total rainfall during the hour with the trapezoidal sum (use hours for the width).

Exercise 16 - Electricity use

A house's power use in kilowatts is $P(0) = 1.2$, $P(2) = 2.0$, $P(4) = 3.4$, $P(6) = 2.6$ at t hours after noon. Estimate the energy used from $t = 0$ to $t = 6$ in kilowatt-hours with a trapezoidal sum.

Exercise 17 - Bracketing the exact value



Left rectangles for x^3 on $[0, 2]$.

For $f(x) = x^3$ on $[0, 2]$ with $n = 4$, find the left and right sums and verify that the exact integral 4 lies between them.

Exercise 18 - Limit of right sums

Show that the right Riemann sum of $f(x) = x$ on $[0, 2]$ with n rectangles is $2 + \frac{2}{n}$ and find its limit.

Exercise 19 - Decreasing function

For the decreasing function $f(x) = \frac{8}{x}$ on $[1, 5]$ with $n = 4$, compute the left and right sums. Which one overestimates?

Exercise 20 - Rectangles and trapezoids

Use geometry to evaluate (a) $\int_1^5 3 \, dx$ and (b) $\int_0^4 (x+2) \, dx$.

Exercise 21 - Highway traffic

hour	0	1	2	3	4
cars/h	120	180	260	200	140

Traffic rate past a toll booth.

The traffic rate past a toll booth is shown in cars per hour, from 6 a.m. to 10 a.m. Estimate the total number of cars with a trapezoidal sum.