

AP Calculus AB - Riemann Sums and the Definite Integral - Test Answer Key

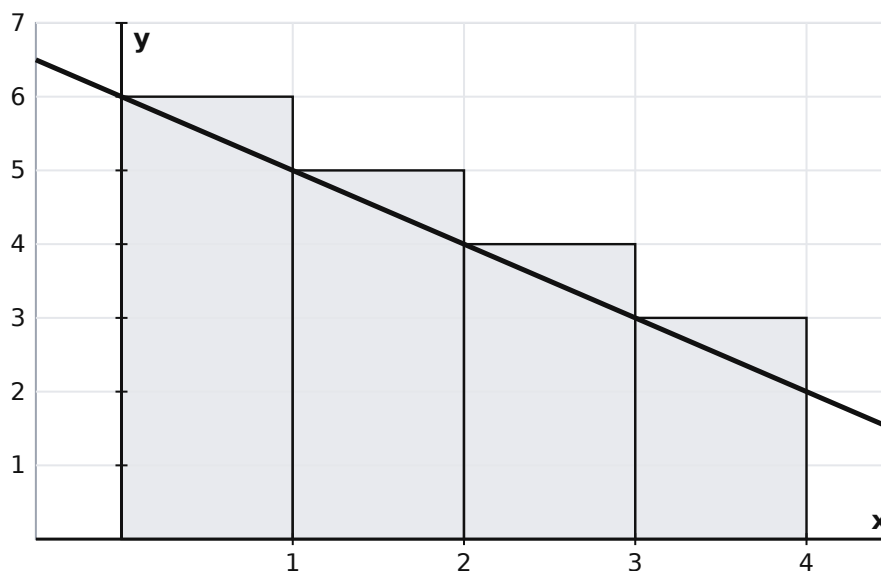
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Use this answer key to score the riemann sums test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 6.1, AP CALC AB 6.2, AP CALC AB 6.3**

Question 1 - Riemann sums for a line [3 pts]

- a. $1(6+5+4+3)=18$.
- b. $1(5+4+3+2)=14$.
- c. $\frac{18+14}{2}=16$. The exact area (trapezoid with parallel sides 6 and 2, width 4) is $\frac{6+2}{2} \cdot 4 = 16$; the trapezoidal sum is exact for a line.



Left rectangles for $f(x) = 6 - x$.

Scoring: 1 point per part.

Question 2 - Table estimate [2 pts]

Right sum: $3(14+15+11+6) = 3(46) = 138$ liters.

The rate rises and then falls, so **we cannot tell from the table alone** whether the right sum is high or low; on the interval where r increases the right sum overestimates and where it decreases it underestimates.

Scoring: 1 point for the sum with units; 1 point for a correct comment about monotonicity.

Question 3 - Over or under [2 pts]

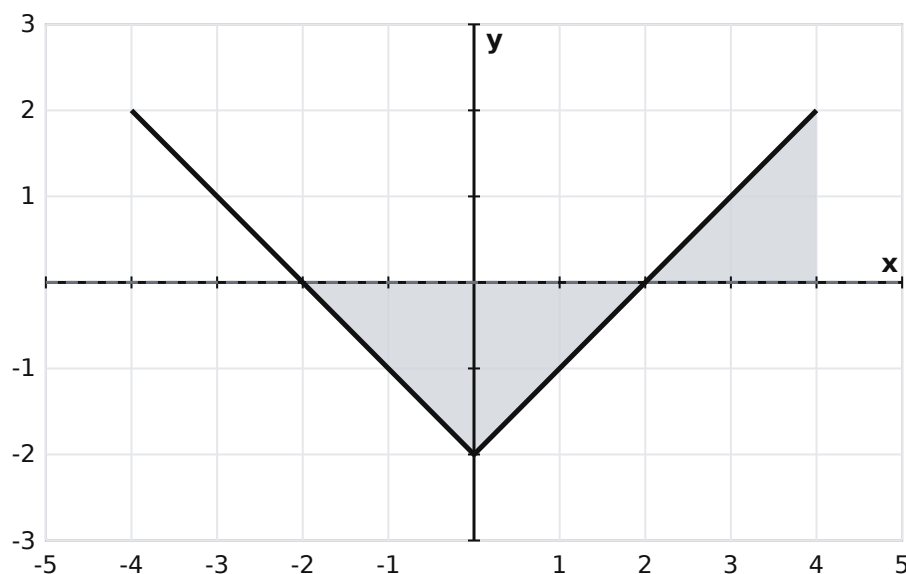
f is decreasing on $[1,3]$. On each subinterval, the right endpoint gives the smallest value, so each rectangle lies below the curve.

Final answer: **underestimate**.

Scoring: 1 point for identifying that f is decreasing; 1 point for the conclusion.

Question 4 - Geometry and integrals [3 pts]

- The region is a triangle below the axis with base 4 and height 2: $-\frac{1}{2}(4)(2) = -4$.
- Add the two triangles above the axis on $[-4,-2]$ and $[2,4]$, each $+2$: $-4+2+2=0$.



Graph of $f(x) = |x| - 2$.

Scoring: 1 point for the first integral; 2 points for the second (1 for the triangles, 1 for the

total).

Question 5 - Properties [3 pts]

- a. $7 + (-3) = 4$.
- b. $-\int_4^9 f = 3$.
- c. $3(4) - 10 = 2$.

Scoring: 1 point per part.

Question 6 - Accumulated change [2 pts]

Left sum: $2(8+6+5) = 38$ gallons.

Final answer: **about 38 gallons** (an overestimate, since the rate is decreasing).

Scoring: 1 point for the sum; 1 point for units.

Question 7 - Sigma and limits [3 pts]

- a. $\sum_{i=1}^3 i^2 \cdot 1 = 1 + 4 + 9 = 14$.
- b. $\Delta x = \frac{4}{n}$ and $x_i = \frac{4i}{n}$ on $[0, 4]$: $\int_0^4 \sqrt{x} \, dx$.

Scoring: (a) 1 point for the setup, 1 point for the value; (b) 1 point.

Question 8 - Displacement and distance [2 pts]

Positive part from 0 to 3: triangle $\frac{1}{2}(3)(3) = 4.5$. Negative part from 3 to 5: $-\frac{1}{2}(2)(2) = -2$.

Displacement **2.5 m**; distance **6.5 m**.

Scoring: 1 point for the displacement; 1 point for the distance.

Total

20 points