

AP Calculus AB - Riemann Sums and the Definite Integral - Test

Common Core AP CALC AB 6.1, AP CALC AB 6.2, AP CALC AB 6.3 · maths-pdf.fr/en/

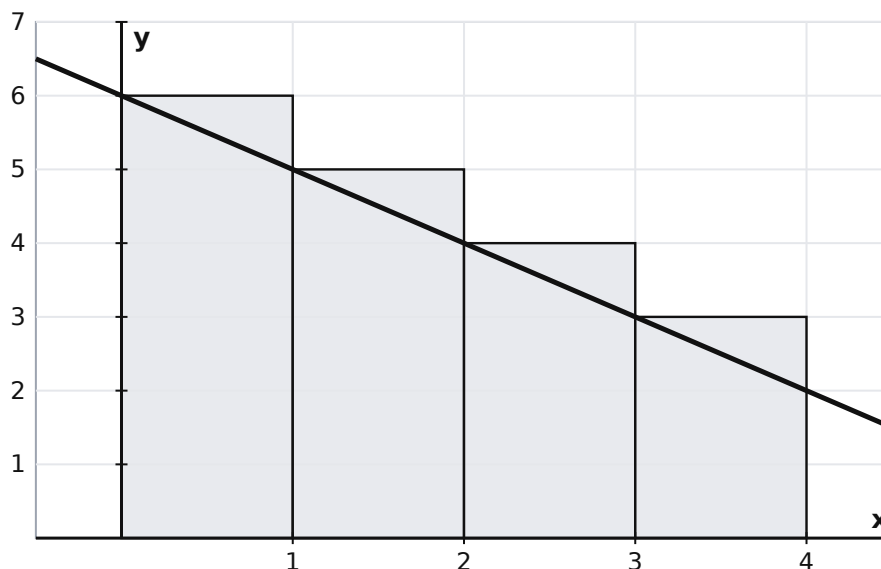
Name: _____ Class: _____ Date: ____/____/____

This test checks Riemann sums, trapezoidal sums, the definite integral as signed area and accumulation of change (AP Calculus AB, 6.1, 6.2, 6.3). It has 8 questions worth 20 points in total. Allow 45 minutes, work without notes, and show all work, since justification earns points on the AP exam.

About this test

- Grade: AP Calculus AB
- Chapter: Riemann Sums and the Definite Integral
- Standards: AP CALC AB 6.1, AP CALC AB 6.2, AP CALC AB 6.3
- Time: 45 minutes
- Total: 20 points

Question 1 - Riemann sums for a line [3 pts]



Left rectangles for $f(x) = 6 - x$.

Let $f(x) = 6 - x$ on $[0, 4]$ with $n = 4$. (a) Find the left sum. (1 point) (b) Find the right sum. (1

point) (c) Find the trapezoidal sum and compare with the exact area. (1 point)

Question 2 - Table estimate [2 pts]

t (min)	0	3	6	9	12
r (L/min)	10	14	15	11	6

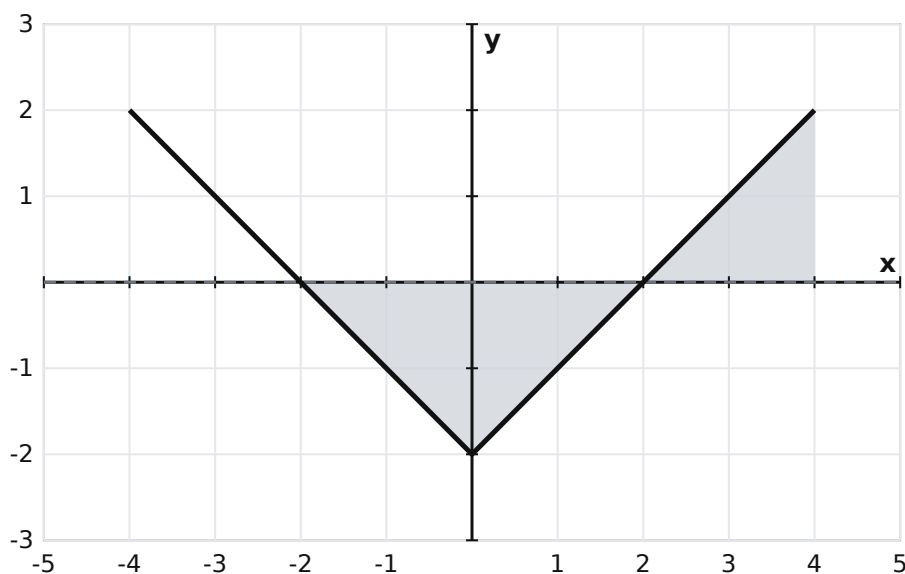
Flow rate of water into a tank.

Water flows into a tank at rate $r(t)$ liters per minute, shown in the table. Use a right sum with 4 subintervals to estimate the total water in 12 minutes. Is it an over- or underestimate? Explain.

Question 3 - Over or under [2 pts]

For $f(x) = \frac{1}{x}$ on $[1, 3]$, explain whether the right sum with 4 equal subintervals overestimates or underestimates the integral.

Question 4 - Geometry and integrals [3 pts]



Graph of $f(x) = |x| - 2$.

Use geometry to evaluate (a) $\int_{-2}^2 (|x| - 2) dx$ and (b) $\int_{-4}^4 (|x| - 2) dx$.

Question 5 - Properties [3 pts]

Given $\int_1^4 f(x) dx = 7$, $\int_4^9 f(x) dx = -3$ and $\int_1^9 g(x) dx = 10$, find (a) $\int_1^9 f(x) dx$, (b) $\int_9^4 f(x) dx$, (c) $\int_1^9 [3f(x) - g(x)] dx$. (1 point each)

Question 6 - Accumulated change [2 pts]

Oil leaks from a tank at $r(t)$ gallons per hour with $r(0) = 8$, $r(2) = 6$, $r(4) = 5$, $r(6) = 2$. Use a left sum with three subintervals to estimate the oil lost during the first 6 hours.

Question 7 - Sigma and limits [3 pts]

(a) Write the right Riemann sum of $f(x) = x^2$ on $[0, 3]$ with $n = 3$ in sigma notation and evaluate it. (2 points) (b) Write $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt{\frac{4i}{n}} \cdot \frac{4}{n}$ as a definite integral. (1 point)

Question 8 - Displacement and distance [2 pts]

A runner's velocity is $v(t) = 3 - t$ m/s for $0 \leq t \leq 5$. Use areas to find the displacement and the total distance.