

AP Calculus AB - Integration by Substitution - Answer Key

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This answer key gives a full worked solution for every problem in the u-substitution worksheet. Compare each step, not only the final answer, because AP graders award credit for a correct setup and clear justification. Final answers are in bold. Redo any problem you missed before moving on to the test.

AP Calculus AB · Common Core State Standards: **AP CALC AB 6.9, AP CALC AB 6.10, AP CALC AB 6.11, AP CALC AB 6.12, AP CALC AB 6.14**

Exercise 1 - A power of a quadratic

Let $u = x^2 + 1$. Then $du = 2x \, dx$.

The integral becomes $\int u^3 du = \frac{u^4}{4}$.

Back-substitute $u = x^2 + 1$ and add the constant.

Final answer: $\frac{(x^2+1)^4}{4} + C$.

Exercise 2 - A square root

Let $u = x^2 + 4$. Then $du = 2x \, dx$.

The integral becomes $\frac{1}{2} \int u^{1/2} du = \frac{1}{2} \cdot \frac{2}{3} u^{3/2}$.

Back-substitute $u = x^2 + 4$ and add the constant.

Final answer: $\frac{1}{3}(x^2+4)^{3/2} + C$.

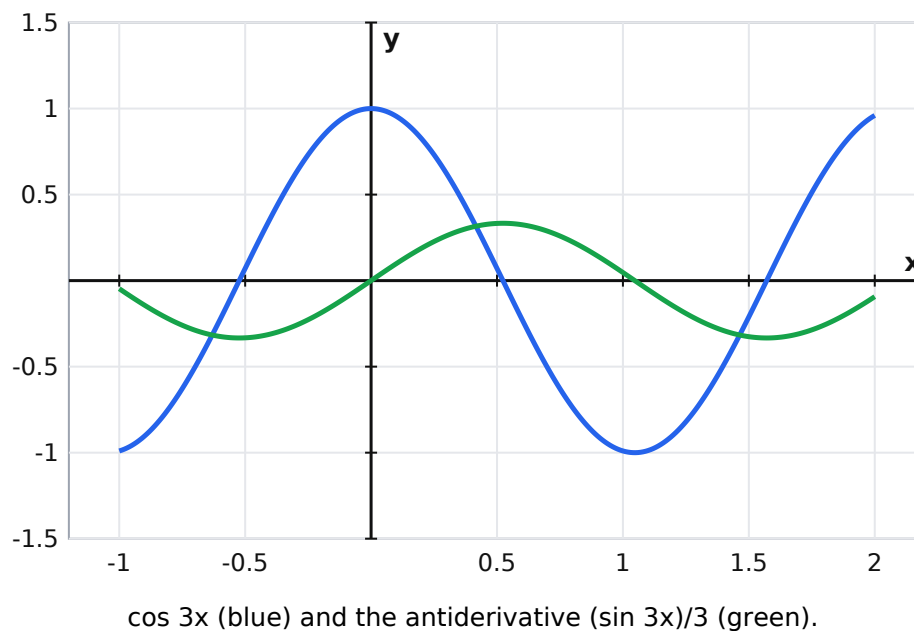
Exercise 3 - Cosine of a multiple

Let $u = 3x$. Then $du = 3 \, dx$.

The integral becomes $\frac{1}{3} \int \cos u \, du = \frac{1}{3} \sin u$.

Back-substitute $u=3x$ and add the constant.

Final answer: $\frac{1}{3}\sin 3x + C$.



Exercise 4 - Exponential of a multiple

Let $u=5x$. Then $du=5 dx$.

The integral becomes $\frac{1}{5}\int e^u du = \frac{1}{5}e^u$.

Back-substitute $u=5x$ and add the constant.

Final answer: $\frac{1}{5}e^{5x} + C$.

Exercise 5 - Exponential with a quadratic

Let $u=x^2$. Then $du=2x dx$.

The integral becomes $\frac{1}{2}\int e^u du = \frac{1}{2}e^u$.

Back-substitute $u=x^2$ and add the constant.

Final answer: $\frac{1}{2}e^{x^2} + C$.

Area: $\left[\frac{1}{2}e^{x^2}\right]_0^1 = \frac{e-1}{2} \approx 0.859$.

Exercise 6 - Sine times a power of cosine

Let $u = \cos x$. Then $du = -\sin x \, dx$.

The integral becomes $-\int u^2 du = -\frac{u^3}{3}$.

Back-substitute $u = \cos x$ and add the constant.

Final answer: $-\frac{\cos^3 x}{3} + C$.

Exercise 7 - Logarithm over x

Let $u = \ln x$. Then $du = \frac{1}{x} \, dx$.

The integral becomes $\int u \, du = \frac{u^2}{2}$.

Back-substitute $u = \ln x$ and add the constant.

Final answer: $\frac{(\ln x)^2}{2} + C$.

Exercise 8 - Reciprocal of a linear function

Let $u = 2x + 3$. Then $du = 2 \, dx$.

The integral becomes $\frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u|$.

Back-substitute $u = 2x + 3$ and add the constant.

Final answer: $\frac{1}{2} \ln|2x + 3| + C$.

Exercise 9 - Quotient with a quadratic

Let $u = x^2 + 1$. Then $du = 2x \, dx$.

The integral becomes $\frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u|$.

Back-substitute $u = x^2 + 1$ and add the constant.

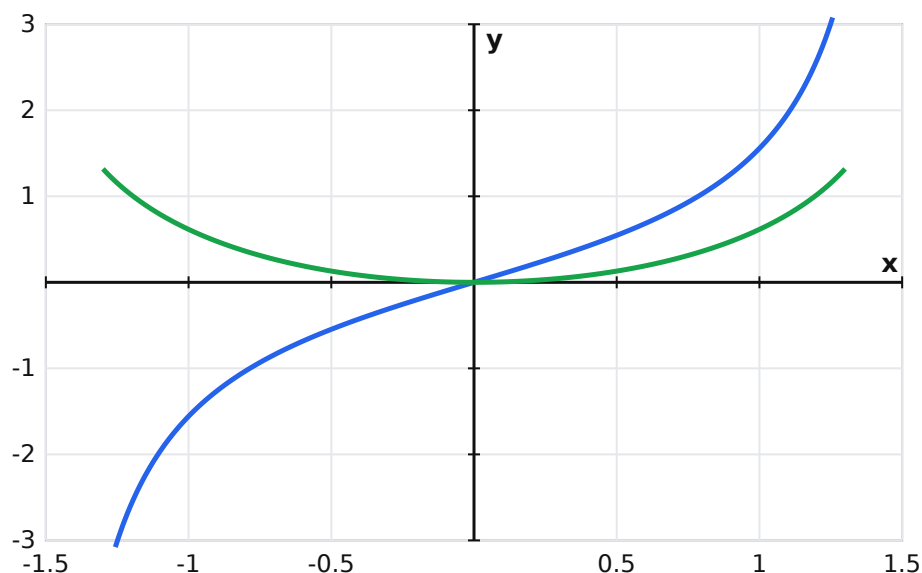
Final answer: $\frac{1}{2} \ln(x^2 + 1) + C$.

Exercise 10 - Integral of tangent

Write $\tan x = \frac{\sin x}{\cos x}$. Let $u = \cos x$, $du = -\sin x \, dx$.

$$-\int \frac{1}{u} du = -\ln|u|.$$

Final answer: $-\ln|\cos x| + C$.



$\tan x$ (blue) and an antiderivative $-\ln|\cos x|$ (green).

Exercise 11 - Exponential over one plus exponential

Let $u = 1 + e^x$. Then $du = e^x \, dx$.

The integral becomes $\int \frac{1}{u} du = \ln|u|$.

Back-substitute $u = 1 + e^x$ and add the constant.

Final answer: $\ln(1 + e^x) + C$.

Exercise 12 - Secant squared of a multiple

Let $u = 2x$. Then $du = 2 \, dx$.

The integral becomes $\frac{1}{2} \int \sec^2 u \, du = \frac{1}{2} \tan u$.

Back-substitute $u = 2x$ and add the constant.

Final answer: $\frac{1}{2}\tan 2x + C$.

Exercise 13 - Inverse tangent form

Let $u = 3x$. Then $du = 3 dx$.

The integral becomes $\frac{1}{3} \int \frac{1}{1+u^2} du = \frac{1}{3} \tan^{-1} u$.

Back-substitute $u = 3x$ and add the constant.

Final answer: $\frac{1}{3} \tan^{-1}(3x) + C$.

Exercise 14 - Cubic inside a power

Let $u = x^3 + 1$. Then $du = 3x^2 dx$.

The integral becomes $\frac{1}{3} \int u^4 du = \frac{1}{3} \cdot \frac{u^5}{5}$.

Back-substitute $u = x^3 + 1$ and add the constant.

Final answer: $\frac{(x^3+1)^5}{15} + C$.

Exercise 15 - Definite integral with changed limits

Let $u = x^2 + 1$, $du = 2x dx$, so $x dx = \frac{1}{2} du$. When $x = 0$, $u = 1$; when $x = 2$, $u = 5$.

$$\frac{1}{2} \int_1^5 u^3 du = \frac{1}{2} \left[\frac{u^4}{4} \right]_1^5 = \frac{625-1}{8} = 78.$$

Final answer: 78.

Exercise 16 - Definite integral, trig

Let $u = \sin x$, $du = \cos x dx$. Limits: $0 \rightarrow 0$ and $\frac{\pi}{2} \rightarrow 1$.

$$\int_0^1 u du = \frac{1}{2}.$$

Final answer: $\frac{1}{2}$.

Exercise 17 - Definite integral, logarithm

Let $u = \ln x$, $du = \frac{1}{x} dx$. Limits: $1 \rightarrow 0$ and $e \rightarrow 1$.

$$\int_0^1 u \, du = \frac{1}{2}.$$

Final answer: $\frac{1}{2}$.

Exercise 18 - Definite integral, rational

Let $u = x^3 + 2$, $du = 3x^2 dx$. Limits: $-1 \rightarrow 1$ and $1 \rightarrow 3$.

$$\frac{1}{3} \int_1^3 \frac{1}{u} du = \frac{1}{3} (\ln 3 - \ln 1) = \frac{\ln 3}{3}.$$

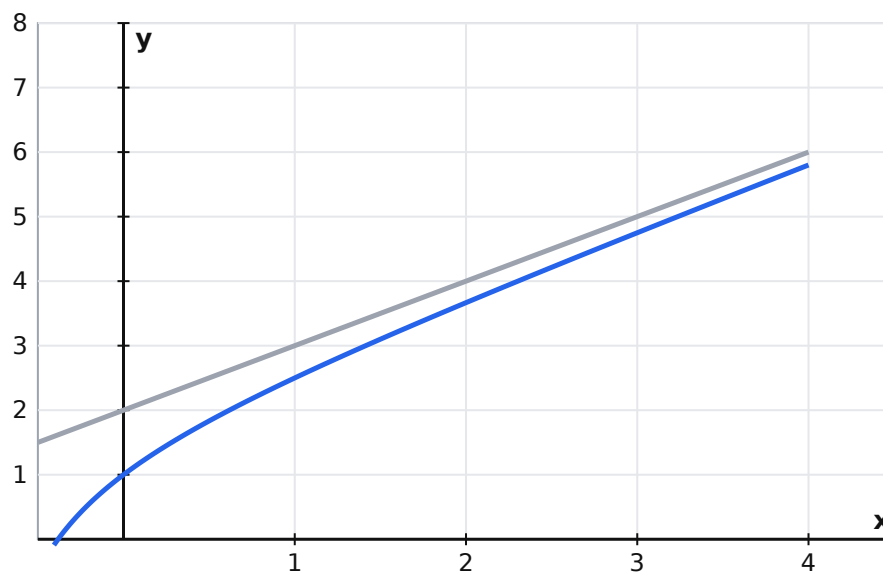
Final answer: $\frac{\ln 3}{3} \approx 0.366$.

Exercise 19 - Long division

Divide: $x^2 + 3x + 1 = (x+1)(x+2) - 1$, so $\frac{x^2 + 3x + 1}{x+1} = x + 2 - \frac{1}{x+1}$.

Integrate term by term.

Final answer: $\frac{x^2}{2} + 2x - \ln|x+1| + C$.



The quotient $x + 2$ is the slant line the graph approaches.

Exercise 20 - Completing the square

Complete the square: $x^2+4x+13=(x+2)^2+9$.

Let $u = x+2$: $\int \frac{1}{u^2+9} du = \frac{1}{3} \tan^{-1} \frac{u}{3}$.

Final answer: $\frac{1}{3} \tan^{-1} \left(\frac{x+2}{3} \right) + C$.

Exercise 21 - Colony growth

Let $u = -0.1t^2$, $du = -0.2t dt$, so $t dt = -5 du$. An antiderivative is $-1000e^{-0.1t^2}$.

$[-1000e^{-0.1t^2}]_0^3 = -1000e^{-0.9} + 1000 = 1000(1 - e^{-0.9}) \approx 593.4$.

Final answer: ≈ 593.4 bacteria.

Exercise 22 - Battery charge

Let $u = -0.5t$, $dt = -2 du$. An antiderivative is $-10e^{-0.5t}$.

$[-10e^{-0.5t}]_0^4 = -10e^{-2} + 10 = 10(1 - e^{-2}) \approx 8.65$.

Final answer: ≈ 8.65 amp-hours.

Exercise 23 - Distance from a velocity

Let $u = t^2 + 16$, $du = 2t dt$, so $3t dt = \frac{3}{2} du$. Limits: $0 \rightarrow 16$ and $3 \rightarrow 25$.

$\frac{3}{2} \int_{16}^{25} u^{1/2} du = \frac{3}{2} \cdot \frac{2}{3} [u^{3/2}]_{16}^{25} = 125 - 64 = 61$.

Final answer: 61 feet.