

AP Calculus AB - Integration by Substitution - Worksheet

Common Core AP CALC AB 6.9, AP CALC AB 6.10, AP CALC AB 6.11, AP CALC AB 6.12, AP CALC AB 6.14 · maths-pdf.fr/en/

Name: _____ Class: _____ Date: ____/____/____

These 23 problems practice integration by u-substitution for indefinite and definite integrals, long division, completing the square and choosing a technique (AP Calculus AB, topics 6.9 to 6.14). The early problems build the basic skill, and each one uses a different function or situation so that no two feel alike.

Later problems use exponential, trig and logarithmic forms, definite integrals with new limits, and situations such as a growing colony and a charging battery. Work in pencil, show every step, and check the answer key only after a full attempt.

AP Calculus AB · Common Core State Standards: **AP CALC AB 6.9, AP CALC AB 6.10, AP CALC AB 6.11, AP CALC AB 6.12, AP CALC AB 6.14**

Exercise 1 - A power of a quadratic

Find $\int 2x(x^2+1)^3 dx$.

Exercise 2 - A square root

Find $\int x\sqrt{x^2+4} dx$.

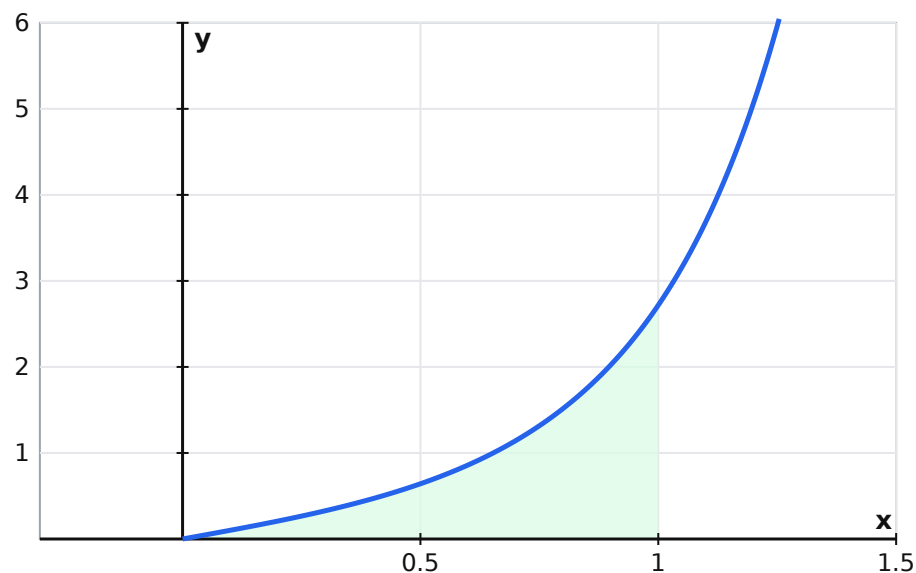
Exercise 3 - Cosine of a multiple

Find $\int \cos(3x) dx$.

Exercise 4 - Exponential of a multiple

Find $\int e^{5x} dx$.

Exercise 5 - Exponential with a quadratic



The area under $x e^{(x^2)}$ from 0 to 1.

Find $\int x e^{x^2} dx$ and use it to find the shaded area $\int_0^1 x e^{x^2} dx$.

Exercise 6 - Sine times a power of cosine

Find $\int \sin x \cos^2 x dx$.

Exercise 7 - Logarithm over x

Find $\int \frac{\ln x}{x} dx$.

Exercise 8 - Reciprocal of a linear function

Find $\int \frac{1}{2x+3} dx$.

Exercise 9 - Quotient with a quadratic

Find $\int \frac{x}{x^2+1} dx$.

Exercise 10 - Integral of tangent

Find $\int \tan x dx$.

Exercise 11 - Exponential over one plus exponential

Find $\int \frac{e^x}{1+e^x} dx$.

Exercise 12 - Secant squared of a multiple

Find $\int \sec^2(2x) dx$.

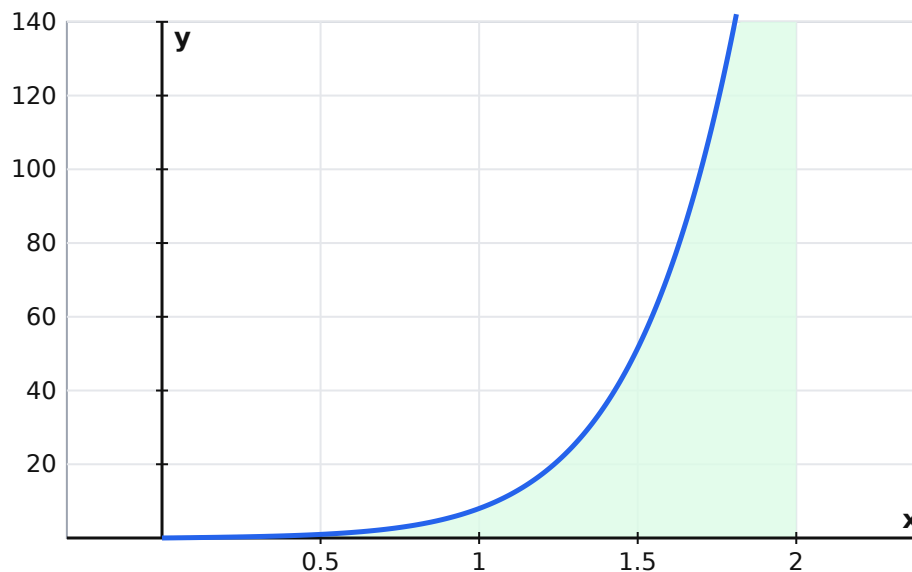
Exercise 13 - Inverse tangent form

Find $\int \frac{1}{1+9x^2} dx$.

Exercise 14 - Cubic inside a power

Find $\int x^2(x^3+1)^4 dx$.

Exercise 15 - Definite integral with changed limits



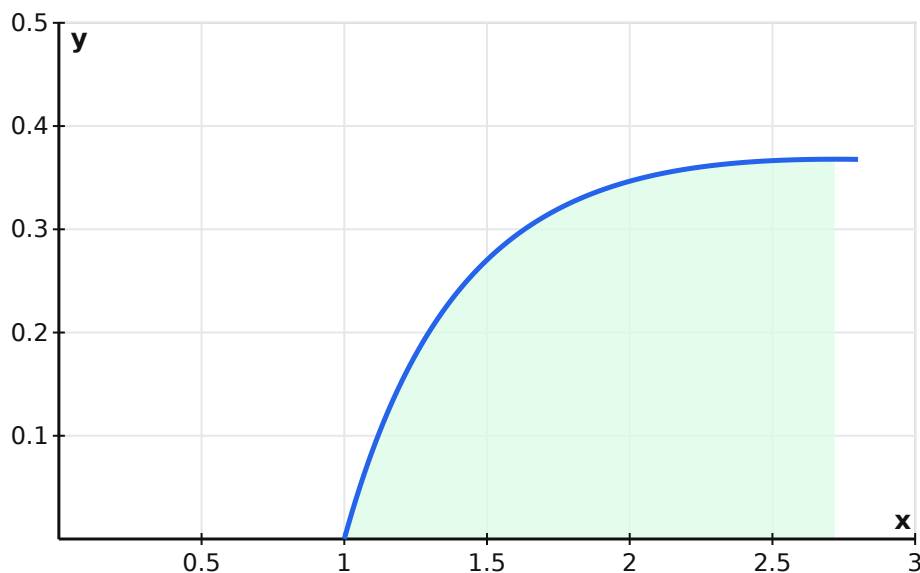
The area under $x(x^2 + 1)^3$ on $[0, 2]$.

Evaluate $\int_0^2 x(x^2+1)^3 dx$ by changing the limits.

Exercise 16 - Definite integral, trig

Evaluate $\int_0^{\pi/2} \sin x \cos x dx$.

Exercise 17 - Definite integral, logarithm



The area under $(\ln x)/x$ from 1 to e .

Evaluate $\int_1^e \frac{\ln x}{x} dx$.

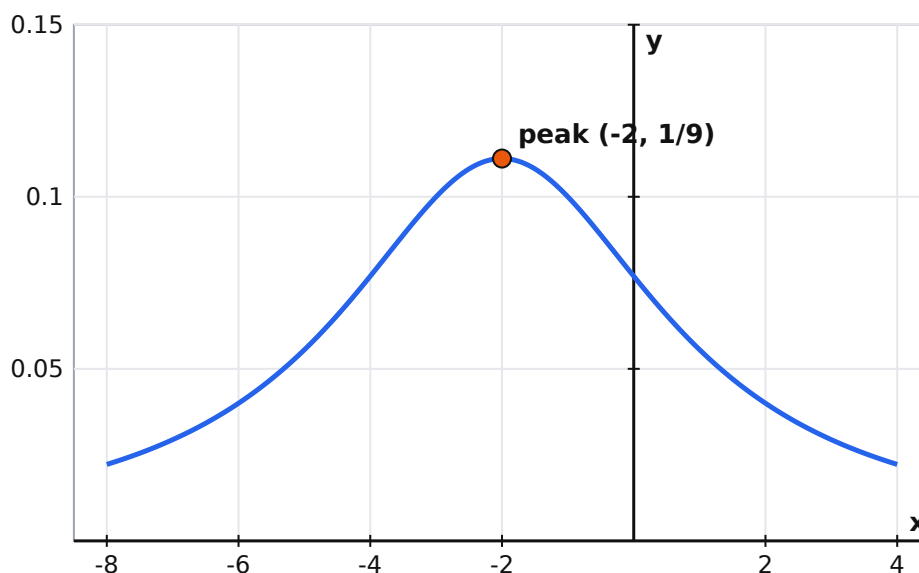
Exercise 18 - Definite integral, rational

Evaluate $\int_{-1}^1 \frac{x^2}{x^3+2} dx$.

Exercise 19 - Long division

Find $\int \frac{x^2+3x+1}{x+1} dx$.

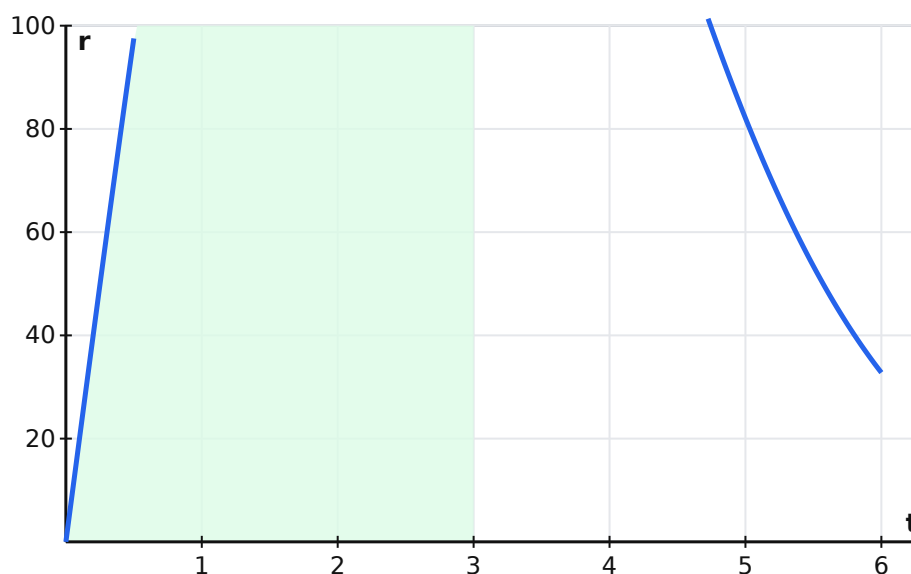
Exercise 20 - Completing the square



The graph of $1/(x^2 + 4x + 13)$.

Find $\int \frac{1}{x^2+4x+13} dx$.

Exercise 21 - Colony growth



Growth rate of a bacteria colony per hour.

A colony grows at the rate $r(t) = 200te^{-0.1t^2}$ bacteria per hour. How many bacteria are added during the first 3 hours?

Exercise 22 - Battery charge

The current into a battery is $I(t) = 5e^{-0.5t}$ amps. Find the charge delivered in the first 4 hours (charge is the integral of current).

Exercise 23 - Distance from a velocity

A drone has velocity $v(t) = 3t\sqrt{t^2+16}$ ft/s. Find the distance traveled from $t=0$ to $t=3$ seconds.