

AP Calculus AB - Integration by Substitution - Test Answer Key

Common Core AP CALC AB 6.9, AP CALC AB 6.10, AP CALC AB 6.11, AP CALC AB 6.12, AP CALC AB 6.14 ·
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Use this answer key to score the u-substitution test out of 20 points. Each solution shows the work and a scoring guide that says how the points are awarded. Give partial credit for a correct setup, a correct derivative or integral, and a correct final statement with units where they apply.

AP Calculus AB · Common Core State Standards: **AP CALC AB 6.9, AP CALC AB 6.10, AP CALC AB 6.11, AP CALC AB 6.12, AP CALC AB 6.14**

Question 1 - Indefinite substitution [2 pts]

Let $u = x^3 - 5$, $du = 3x^2 dx$. $\int u^4 du = \frac{u^5}{5}$.

Final answer: $\frac{(x^3 - 5)^5}{5} + C$.

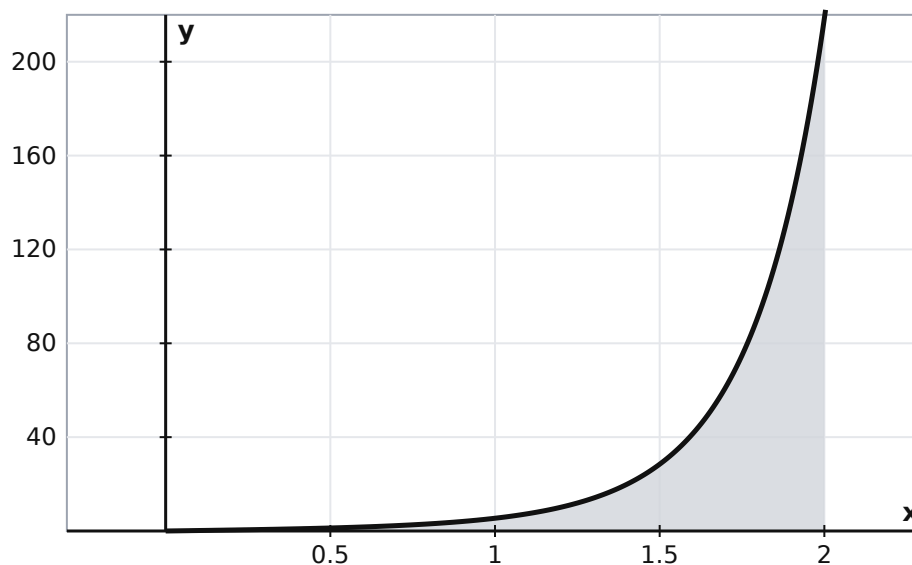
Scoring: 1 point for u and du ; 1 point for the final antiderivative.

Question 2 - Definite substitution [3 pts]

Let $u = x^2$, $du = 2x dx$. Limits: $0 \rightarrow 0$, $2 \rightarrow 4$.

$\int_0^4 e^u du = e^4 - 1 \approx 53.6$.

Final answer: $e^4 - 1$.



The graph of $f(x) = 2x e^{(x^2)}$.

Scoring: 1 point for u , du and limits; 1 point for the antiderivative; 1 point for the value.

Question 3 - Exponential substitution [2 pts]

Let $u = 3x+1$, $dx = \frac{1}{3}du$. The integral is $\frac{1}{3}e^u$.

Final answer: $\frac{1}{3}e^{3x+1} + C$.

Scoring: 1 point for the substitution; 1 point for the result.

Question 4 - Trigonometric substitution [3 pts]

Let $u = \sin x$, $du = \cos x dx$. Limits: $0 \rightarrow 0$, $\frac{\pi}{2} \rightarrow 1$.

$$\int_0^1 u^3 du = \frac{1}{4}.$$

Final answer: $\frac{1}{4}$.

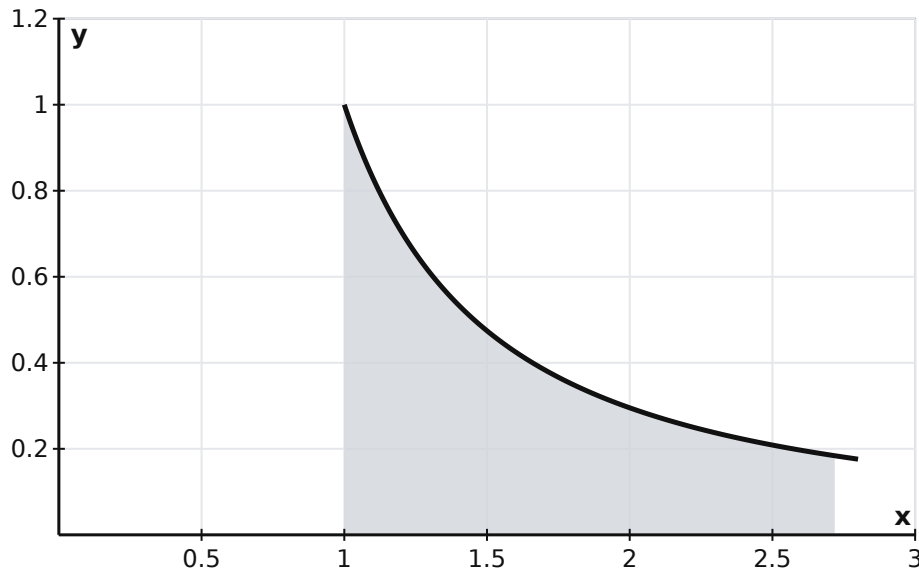
Scoring: 1 point for u and du ; 1 point for the limits; 1 point for the value.

Question 5 - Logarithm form [3 pts]

Let $u = 1 + \ln x$, $du = \frac{1}{x} dx$. Limits: $1 \rightarrow 1$, $e \rightarrow 2$.

$$\int_1^2 \frac{1}{u} du = \ln 2.$$

Final answer: $\ln 2 \approx 0.693$.



The graph of $f(x) = 1/(x(1 + \ln x))$.

Scoring: 1 point for u and du ; 1 point for the limits; 1 point for the value.

Question 6 - Long division [2 pts]

Divide: $x^2+5=(x+2)(x-2)+9$, so $\frac{x^2+5}{x+2} = x-2 + \frac{9}{x+2}$.

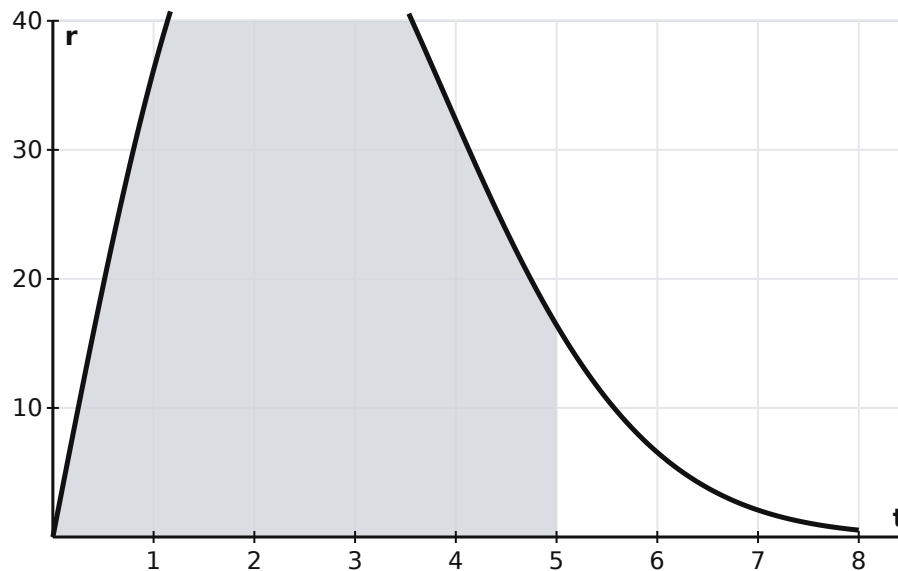
Final answer: $\frac{x^2}{2} - 2x + 9\ln|x+2| + C$.

Scoring: 1 point for the division; 1 point for the antiderivative.

Question 7 - Inflow of water [3 pts]

a. $\int_0^5 40te^{-t^2/10} dt$.

b. Let $u = -\frac{t^2}{10}$, $t dt = -5 du$. An antiderivative is $-200e^{-t^2/10}$;
 $\left[-200e^{-t^2/10}\right]_0^5 = 200(1 - e^{-2.5}) \approx 183.6$ gallons.



Inflow rate in gallons per minute.

Scoring: 1 point for the integral; 1 point for the antiderivative; 1 point for the value.

Question 8 - Choosing a technique [2 pts]

- u-substitution with $u = x^2$: $\frac{1}{2}\sin(x^2) + C$.
- Split the fraction (division): $x + \frac{1}{x}$, so $\frac{x^2}{2} + \ln|x| + C$.

Scoring: 1 point per part.

Total

20 points