

AP Statistics - Binomial and Geometric Distributions - Lesson

Common Core AP STAT 4.10, AP STAT 4.11, AP STAT 4.12, HSS-MD.A.3, HSS-CP.B.9 · maths-pdf.fr/en/

A binomial random variable counts successes in a fixed number n of independent trials with the same success probability p ; use the BINS conditions to check. Its mean is np . A geometric random variable counts the trials until the first success, with $P(X = n) = (1-p)^{(n-1)} p$ and mean $1/p$.

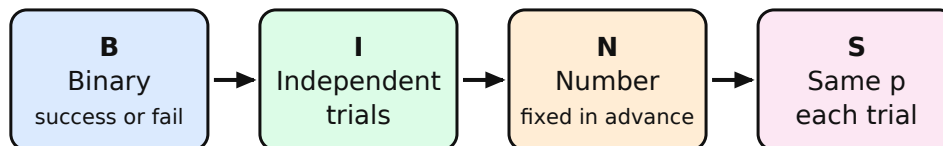
AP Statistics · Common Core State Standards: **AP STAT 4.10, AP STAT 4.11, AP STAT 4.12, HSS-MD.A.3, HSS-CP.B.9**

Key facts

- Binomial conditions: Binary, Independent, fixed Number, Same probability.
- Binomial: mean = np , standard deviation = $\sqrt{np(1-p)}$; use `binompdf` and `binomcdf`.
- At least k successes: $1 - \text{binomcdf}(n, p, k - 1)$.
- Geometric: trials until the first success; $P(X = n) = (1-p)^{(n-1)} p$; mean $1/p$.
- Without replacement, treat trials as independent only if n is at most 10% of the population.

What is a binomial distribution?

A **binomial setting** has a fixed number n of independent trials, each with two outcomes (success or failure) and the same probability of success p . The random variable X counts the successes. Check the conditions with BINS.

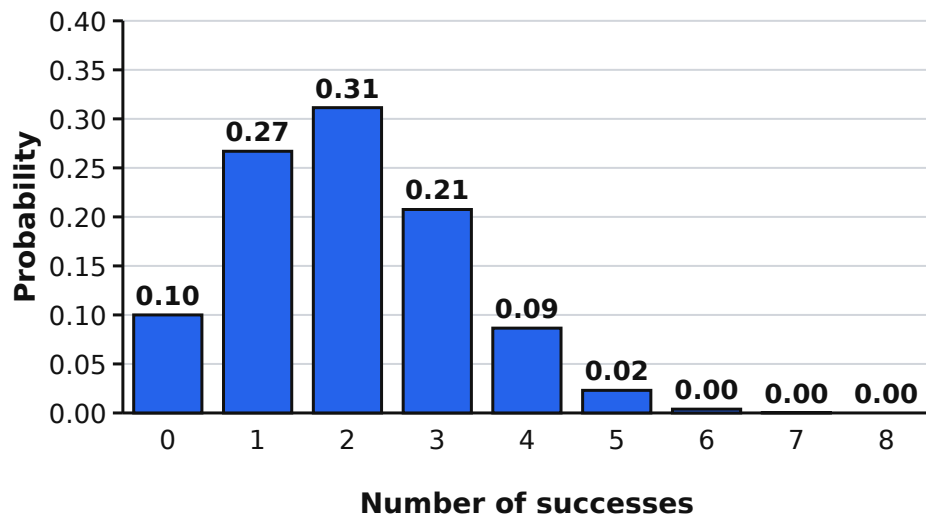


All four BINS conditions must hold.

Example: a student guesses on $n=8$ multiple-choice questions with 4 options. Then $p=0.25$, and X is the number of correct guesses.

$$P(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

For exactly 2 correct: $P(X = 2) = \frac{8!}{2!6!} (0.25)^2 (0.75)^6 = 28(0.0625)(0.17798) = 0.3115$.



Binomial with $n = 8$ and $p = 0.25$: skewed right, mean 2.

The mean and standard deviation of a binomial are $\mu_X = np$ and $\sigma_X = \sqrt{np(1-p)}$. Here $\mu_X = 8(0.25) = 2$ and $\sigma_X = \sqrt{8(0.25)(0.75)} = 1.225$.

How do you compute binomial probabilities?

Question	Calculator command
$P(X = k)$	<code>binompdf(n, p, k)</code>
$P(X \leq k)$	<code>binomcdf(n, p, k)</code>
$P(X \geq k)$	<code>1 - binomcdf(n, p, k - 1)</code>
Geometric $P(X = k)$	<code>geompdf(p, k)</code>
Geometric $P(X \leq k)$	<code>geomcdf(p, k)</code>

Calculator commands.

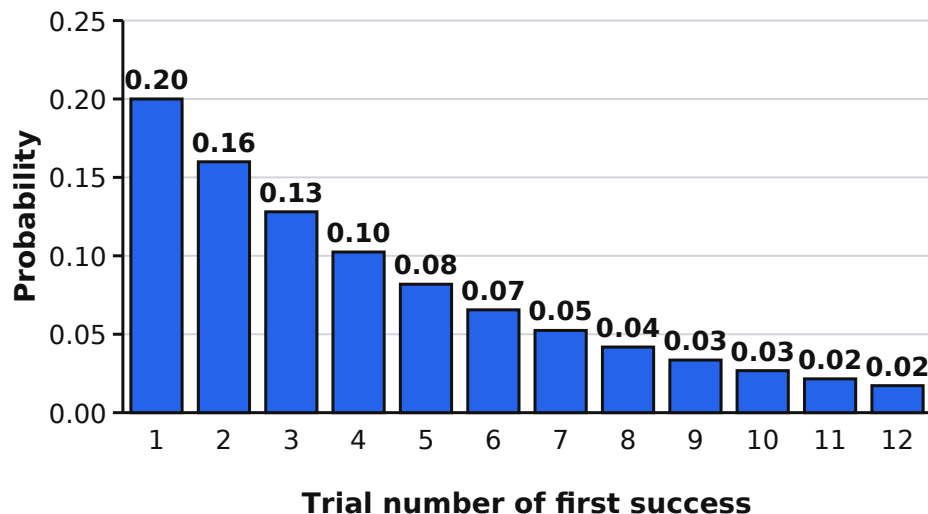
1. State the distribution: X is binomial with $n = 8$ and $p = 0.25$.
2. Write the event, such as $P(X \geq 3)$.
3. Use `binompdf` for one value and `binomcdf` for "at most." For "at least," use the complement.
4. Answer in context.

$$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.6785 = 0.3215.$$

Interpreting a binomial answer matters as much as computing it. If the chance of 4 or more correct guesses is about 0.114, then a student who guesses on every question will reach that score on roughly one quiz in nine. A result with probability below 5% is usually called unusual, and the count of successes should be described with the context, for example "the number of correct answers out of 8 questions." The shape depends on p : it is skewed right when p is small, symmetric when $p = 0.5$, and skewed left when p is large.

What is a geometric distribution?

A **geometric setting** has independent trials with the same probability of success p , repeated until the first success. X counts the trials needed. There is no fixed n .



Geometric with $p = 0.2$: always skewed right, with the highest bar at 1.

$$P(X = n) = (1-p)^{n-1} p, \quad \mu_X = \frac{1}{p}$$

If 20% of cereal boxes have a prize ($p = 0.2$), the chance that the first prize appears in the 4th box is $0.8^3(0.2) = 0.1024$. You expect to open $\frac{1}{0.2} = 5$ boxes on average.

Questions about binomial and geometric distributions

What does BINS stand for?

BINS stands for Binary outcomes (success or failure), Independent trials, a fixed Number of trials, and the Same probability of success on each trial. All four must hold for a binomial model.

How do I find 'at least' probabilities?

Use the complement. For $P(X \geq 3)$ in n trials, compute $1 - P(X \leq 2) = 1 - \text{binomcdf}(n, p, 2)$. Doing this with the calculator avoids adding many terms.

What is the difference between binomial and geometric?

A binomial counts successes in a fixed number of trials. A geometric counts the number of trials until the first success, so its number of trials is not fixed.

Can I use a binomial when sampling without replacement?

Yes if the sample is less than 10% of the population. Then the probability of success changes very little, so the trials are approximately independent.

Why is the geometric distribution always skewed right?

The probabilities start at p for the first trial and keep decreasing by a factor of $(1 - p)$ each trial. The long tail extends to the right.