

AP Statistics - Chi-Square Tests - Lesson

Common Core AP STAT 8.1, AP STAT 8.2, AP STAT 8.3, AP STAT 8.4, AP STAT 8.5, AP STAT 8.6, AP STAT 8.7 · maths-pdf.fr/en/

A chi-square test compares observed counts with the counts expected under the null hypothesis, using the sum of $(O - E)^2/E$ and a p-value. Goodness-of-fit has $df = \text{categories} - 1$; a two-way table has $E = \text{row total} \times \text{column total} / \text{grand total}$ and $df = (r - 1)(c - 1)$. Expected counts must be at least 5.

AP Statistics · Common Core State Standards: **AP STAT 8.1, AP STAT 8.2, AP STAT 8.3, AP STAT 8.4, AP STAT 8.5, AP STAT 8.6, AP STAT 8.7**

Key facts

- chi-square = sum of $(O - E)^2 / E$; right-tailed p-value; distribution skewed right.
- Goodness-of-fit: one categorical variable; $df = \text{number of categories} - 1$.
- Two-way table: $E = \text{row total} \times \text{column total} / \text{grand total}$; $df = (r - 1)(c - 1)$.
- Conditions: random sample or random assignment, and all expected counts at least 5.
- Homogeneity: compare distributions across groups; independence: two variables in one sample.

What is a chi-square test?

A **chi-square test** compares observed counts in categories with the counts we would expect if a claim were true. The test statistic adds up the squared differences, scaled by the expected counts:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Large values of χ^2 mean the data are far from what the null hypothesis predicts. The p-value is the area to the right of χ^2 under a chi-square distribution with the appropriate degrees of freedom. These distributions are skewed right.

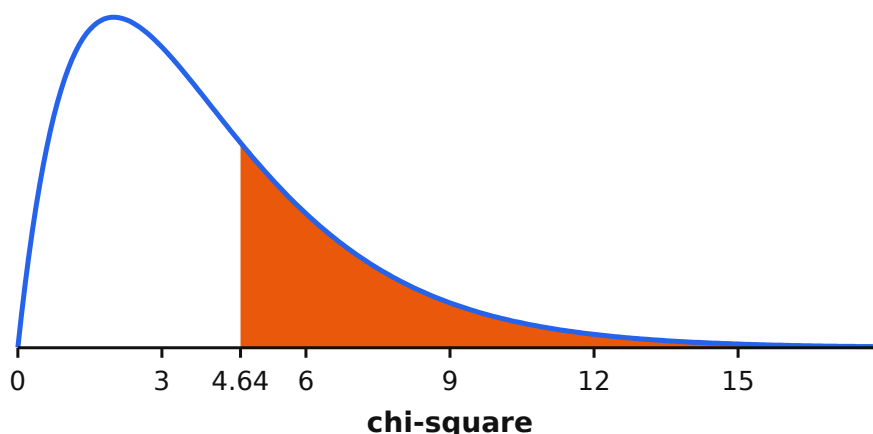
How does a goodness-of-fit test work?

A goodness-of-fit test checks whether one categorical variable has a claimed distribution. Example: a manager claims that late arrivals are equally likely on each of 5 weekdays. In 250 late arrivals, the counts are 58, 48, 44, 42, 58.

Category	Observed	Expected
Mon	58	50
Tue	48	50
Wed	44	50
Thu	42	50
Fri	58	50

Goodness-of-fit setup: expected counts are 250 times each hypothesized proportion.

- H_0 : the proportion of late arrivals is 0.20 on each weekday. H_a : at least one proportion differs.
- **Conditions:** random sample, and all expected counts at least 5 (here 50 each).
- $\chi^2 = \frac{(58-50)^2}{50} + \frac{(48-50)^2}{50} + \frac{(44-50)^2}{50} + \frac{(42-50)^2}{50} + \frac{(58-50)^2}{50} = 4.64$ with $df = 5 - 1 = 4$.



The chi-square distribution is skewed right; the p-value is the area to the right of 4.64 (about 0.326).

The p-value is 0.3263, greater than 0.05, so we fail to reject H_0 . There is not convincing evidence that late arrivals are unevenly distributed across weekdays.

How do you test two categorical variables?

For a two-way table, the expected count in each cell is $E = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$, and $df = (r-1)(c-1)$. The same formula gives χ^2 . Use a test for **homogeneity** when comparing the distribution of one variable across several populations or treatments, and a test for **independence** when you classify one sample by two variables.

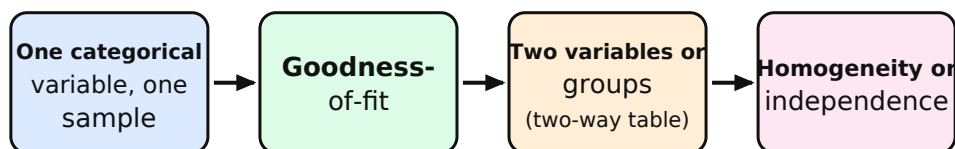
	Library	Home	Cafe	Total
Sophomore	30	45	25	100
Senior	50	35	15	100
Total	80	80	40	200

Observed counts of study location by class (two-way table).

	Library	Home	Cafe	Total
Sophomore	30 (40.0)	45 (40.0)	25 (20.0)	100
Senior	50 (40.0)	35 (40.0)	15 (20.0)	100
Total	80	80	40	200

Observed (expected) counts. Expected = row total times column total over grand total.

Here $\chi^2 = 8.75$, $df = (2-1)(3-1) = 2$, and the p-value is 0.0126. Since this is less than 0.05, we reject H_0 : there is convincing evidence of an association between class and study location (the seniors prefer the library more than the sophomores do).



Choosing the test.

Questions about chi-square tests

What does a large chi-square statistic mean?

It means the observed counts differ greatly from the expected counts under H_0 . The larger the statistic, the smaller the p-value and the stronger the evidence against H_0 .

What is the condition for using a chi-square test?

The data must come from a random sample or randomized experiment, and every expected count must be at least 5. Observed counts may be small, but expected counts may not.

What is the difference between homogeneity and independence?

Homogeneity compares the distribution of one categorical variable across several separate populations or treatments. Independence asks whether two categorical variables are related in a single sample. The calculation is identical; the wording of the hypotheses differs.

How do I find the degrees of freedom?

For goodness-of-fit, df is the number of categories minus 1. For a two-way table, df is (number of rows - 1) times (number of columns - 1).

Which cells contribute the most to chi-square?

Compute $(O - E)^2 / E$ for each cell. The largest contributions show where the data differ most from H_0 , which is useful in your conclusion.