

AP Statistics - Chi-Square Tests - Answer Key

Common Core AP STAT 8.1, AP STAT 8.2, AP STAT 8.3, AP STAT 8.4, AP STAT 8.5, AP STAT 8.6, AP STAT 8.7 · maths-pdf.fr/en/

Each solution lists hypotheses, expected counts, conditions, the chi-square statistic with df, the p-value, and a conclusion in context. P-values come from the chi-square distribution and are rounded to four decimals.

AP Statistics · Common Core State Standards: **AP STAT 8.1, AP STAT 8.2, AP STAT 8.3, AP STAT 8.4, AP STAT 8.5, AP STAT 8.6, AP STAT 8.7**

Exercise 1 - Expected counts for GOF

$$0.30(200) = 60, 0.30(200) = 60, 0.20(200) = 40, 0.20(200) = 40.$$

Red 60, blue 60, green 40, yellow 40.

Exercise 2 - Degrees of freedom

$$(a) 6-1=5. (b) (3-1)(4-1)=6. (c) (2-1)(2-1)=1.$$

5; 6; 1.

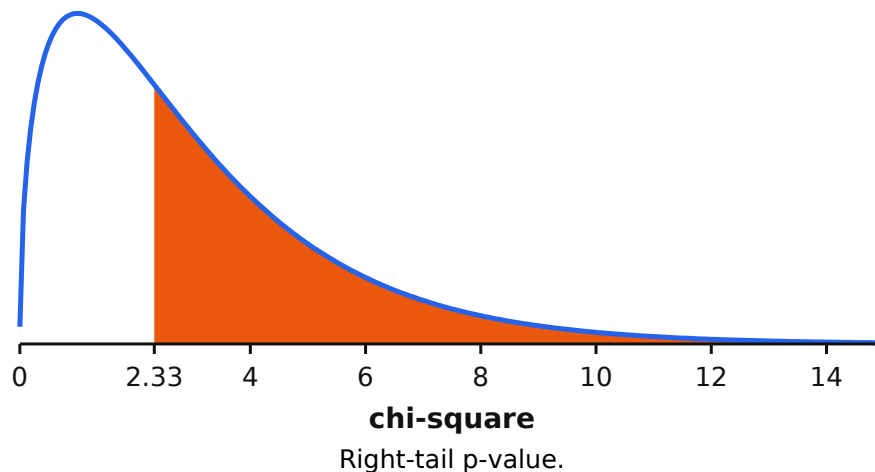
Exercise 3 - Compute chi-square

$$\chi^2 = \frac{(68-60)^2}{60} + \frac{(56-60)^2}{60} + \frac{(42-40)^2}{40} + \frac{(34-40)^2}{40}$$

$$= 1.067 + 0.267 + 0.100 + 0.900 = 2.333.$$

chi-square = 2.333, df = 3.

Exercise 4 - p-value



$$p\text{-value} = P(\chi^2_3 > 2.333) = 0.5062.$$

0.5062 > 0.05: fail to reject H₀. There is not convincing evidence that the color mix differs from the company's claim.

Exercise 5 - Largest contribution

Contributions: red 1.067, blue 0.267, green 0.100, yellow 0.900.

Red contributes the most (1.067): there were 8 more red candies than expected.

Exercise 6 - Hypotheses

A die has 6 faces.

H₀: the probability of each face is 1/6. H_a: at least one face has a probability different from 1/6.

Exercise 7 - Check conditions

Expected counts: 14, 12, 8, 4, 2.

No: two expected counts (4 and 2) are less than 5.

Exercise 8 - Fair die

Category	Observed	Expected
1	14	20
2	21	20
3	17	20
4	22	20
5	19	20
6	27	20

120 rolls of a die.

Expected 20 each (all at least 5). $\chi^2=5.00$, $df=5$, $p\text{-value}=0.4159$.

Fail to reject H_0 : not convincing evidence that the die is unfair.

Exercise 9 - Expected counts in a table

$E = \frac{100 \times 80}{200} = 40$ (row total 100 for sophomores, column total 80 for the library).

40.

Exercise 10 - Chi-square for the table

	Library	Home	Cafe	Total
Sophomore	30 (40.0)	45 (40.0)	25 (20.0)	100
Senior	50 (40.0)	35 (40.0)	15 (20.0)	100
Total	80	80	40	200

Observed (expected) counts. Expected = row total times column total over grand total.

Contributions: 2.50, 0.62, 1.25, 2.50, 0.62, 1.25.

$\chi^2=8.750$, $df=2$.

chi-square = 8.750; df = 2.

Exercise 11 - Conclusion for the table

$$0.0126 < 0.05.$$

Reject H_0 . There is convincing evidence of an association between class (sophomore or senior) and preferred study location.

Exercise 12 - Homogeneity

Homogeneity test: H_0 : the distribution of opinion is the same in all regions. Expected counts: 43.3, 56.7, 43.3, 56.7, 43.3, 56.7 (all at least 5).

$$\chi^2 = 8.82, df = 2, p\text{-value} = 0.0121.$$

Reject H_0 : convincing evidence that opinion differs among regions.

Exercise 13 - Independence or homogeneity?

(a) One sample, two variables: independence. (b) Separate samples from three populations, one variable: homogeneity.

(a) chi-square test for independence; (b) chi-square test for homogeneity.

Exercise 14 - Missing expected count

$$\text{Expected: } 30, 30, 40. \quad \chi^2 = \frac{0^2}{30} + \frac{(25-30)^2}{30} + \frac{(45-40)^2}{40} = 0 + 0.833 + 0.625 = 1.458.$$

chi-square = 1.458 with $df = 2$; the p-value is 0.4823, which gives no convincing evidence against the null proportions.

Exercise 15 - Wrong test

The same students are measured twice, so the data are not independent.

Chi-square assumes independent observations; these are paired data, so the test is not appropriate.

Exercise 16 - Small expected counts

Two expected counts are below 5.

No; combine categories if reasonable or collect more data.

Exercise 17 - 2 by 2 table

Expected counts: 28.0, 42.0, 52.0, 78.0 (all at least 5). $\chi^2 = 36.63$, $df = 1$, $p\text{-value} = 0.000000$.

Reject H_0 : convincing evidence of an association between age group and streaming.

Exercise 18 - Which procedure?

(a) Two-sample t. (b) Chi-square goodness-of-fit (or one-proportion z). (c) One-proportion z interval. (d) Chi-square test for independence.

(a) two-sample t test; (b) goodness-of-fit; (c) one-proportion z interval; (d) chi-square independence.

Exercise 19 - Multi-step

Expected 22.5 each. Contributions: 2.500, 8.100, 0.100, 2.500. $\chi^2 = 13.20$, $df = 3$, $p\text{-value} = 0.0042$.

Reject H_0 ; convincing evidence that seasons are not equally popular. Summer contributes the most.

Exercise 20 - Interpret the p-value

The p-value is computed assuming no association.

If class level and car type were independent in the population, there would be about a 2% chance of getting a chi-square statistic at least this large in a random sample.