

AP Statistics - Conditional Probability and Independence - Lesson

Common Core AP STAT 4.5, AP STAT 4.6, HSS-CP.A.2, HSS-CP.A.3, HSS-CP.A.4, HSS-CP.B.6, HSS-CP.B.8 · maths-pdf.fr/en/

Conditional probability $P(A \text{ given } B) = P(A \text{ and } B)/P(B)$ is the probability of A when B is known to have occurred. The general multiplication rule is $P(A \text{ and } B) = P(B) P(A \text{ given } B)$. Events are independent if $P(A \text{ given } B) = P(A)$, equivalently $P(A \text{ and } B) = P(A) P(B)$. Tree diagrams and two-way tables organize these calculations.

AP Statistics · Common Core State Standards: **AP STAT 4.5, AP STAT 4.6, HSS-CP.A.2, HSS-CP.A.3, HSS-CP.A.4, HSS-CP.B.6, HSS-CP.B.8**

Key facts

- $P(A | B) = P(A \text{ and } B) / P(B)$: restrict to the outcomes where B occurred.
- General multiplication rule: $P(A \text{ and } B) = P(B) P(A | B)$.
- Independent: $P(A | B) = P(A)$, so $P(A \text{ and } B) = P(A) P(B)$.
- Mutually exclusive events with positive probabilities are never independent.
- Tree diagrams multiply along branches and add the paths that fit the event.

What is conditional probability?

The **conditional probability** of A given B , written $P(A|B)$, is the probability that A occurs when we know that B has occurred:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Group	Under 30	30 or older	Total
Streams daily	48	22	70
Does not	32	98	130
Total	80	120	200

200 adults by age group and daily streaming.

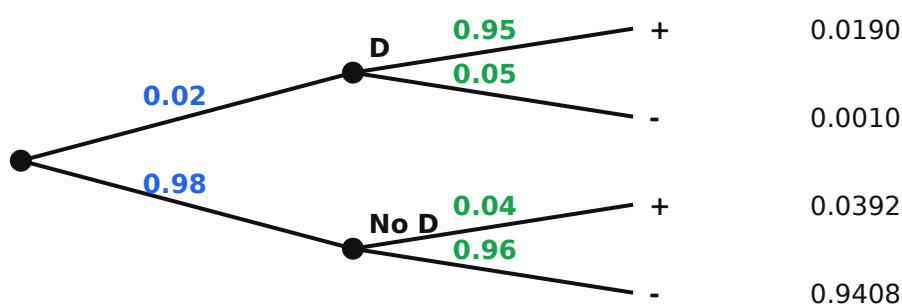
In the table, $P(\text{streams}|\text{under } 30) = \frac{48}{80} = 0.60$. We restrict attention to the 80 adults under 30 and ask what fraction of them stream daily. Compare with $P(\text{streams}) = \frac{70}{200} = 0.35$.

What is the general multiplication rule?

Rule	Formula
Conditional	$P(A B) = P(A \text{ and } B) / P(B)$
Multiplication	$P(A \text{ and } B) = P(B) \times P(A B)$
Independent	$P(A \text{ and } B) = P(A) \times P(B)$

The three key formulas.

Rearranging the conditional probability formula gives $P(A \text{ and } B) = P(B) \cdot P(A|B)$. A **tree diagram** applies this rule along each branch: multiply the probabilities along a path to get the probability of that outcome.



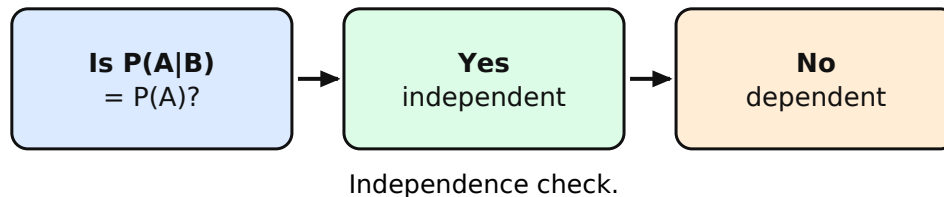
Tree diagram for a medical test.

The probability of a positive test is $0.0190 + 0.0392 = 0.0582$. Given a positive test, the chance of actually having the disease is $P(D|+) = \frac{0.0190}{0.0582} \approx 0.326$, much lower than the 0.95 sensitivity, because the disease is rare.

Worked example. Suppose 60% of students at a school take the bus, and 25% of bus riders also play a sport after school. Then the probability that a randomly chosen student rides the bus and plays a sport is $0.60 \times 0.25 = 0.15$. The same value comes from a table of 1,000 students: 600 ride the bus, 150 of those play a sport, and $\frac{150}{1000} = 0.15$. Both methods agree, so use whichever representation the problem gives you.

What does independence mean?

Events A and B are **independent** if knowing that one occurred does not change the probability of the other: $P(A|B) = P(A)$. Equivalently, $P(A \text{ and } B) = P(A) P(B)$.



In the streaming table, 0.60 is not equal to 0.35, so streaming and age group are not independent. Do not confuse independent with mutually exclusive: events with positive probabilities that are mutually exclusive are always dependent, because if one occurs the other cannot.

1. Write the event in "A given B" form and identify the group B.
2. Restrict to B (use the row or column total from the table or the branch from the tree).
3. Compute the fraction or product of branch probabilities.
4. Interpret the probability in context.

Questions about conditional probability and independence

What does 'given' mean in probability?

'Given B' means we already know B happened, so we look only at the outcomes in B. In a two-way table, it means using the row or column total of B as the denominator.

How can I check whether two events are independent?

Compute $P(A | B)$ and compare it with $P(A)$, or check whether $P(A \text{ and } B)$ equals $P(A)$ times $P(B)$. If they match, the events are independent; if not, they are dependent.

Are mutually exclusive events independent?

No, not if both have positive probability. If A occurs, B cannot, so knowing A changes the probability of B to 0, which is not $P(B)$.

When can I multiply probabilities?

Multiply $P(A)$ and $P(B)$ only when A and B are independent. Otherwise use the general multiplication rule $P(A) \text{ times } P(B | A)$.

Why is $P(D | +)$ different from $P(+ | D)$?

They answer different questions. $P(+ | D)$ is the test's sensitivity, while $P(D | +)$ depends also on how common the disease is and how often healthy people test positive.