

AP Statistics - Conditional Probability and Independence - Answer Key

Common Core AP STAT 4.5, AP STAT 4.6, HSS-CP.A.2, HSS-CP.A.3, HSS-CP.A.4, HSS-CP.B.6, HSS-CP.B.8 · maths-pdf.fr/en/

Each solution identifies the condition, shows the restricted denominator or the branch multiplication, and gives an exact fraction with a decimal. Independence checks compare $P(A \text{ given } B)$ with $P(A)$.

AP Statistics · Common Core State Standards: **AP STAT 4.5, AP STAT 4.6, HSS-CP.A.2, HSS-CP.A.3, HSS-CP.A.4, HSS-CP.B.6, HSS-CP.B.8**

Exercise 1 - Conditional from a table

Group	Under 30	30 or older	Total
Streams daily	48	22	70
Does not	32	98	130
Total	80	120	200

200 adults by age group and daily streaming.

First: $\frac{48}{80} = 0.60$. Second: $\frac{48}{70} \approx 0.686$.

0.60 and about 0.686; the conditions are different.

Exercise 2 - Use the formula

$$P(A|B) = \frac{0.2}{0.4} = 0.5. \quad P(B|A) = \frac{0.2}{0.5} = 0.4.$$

0.5 and 0.4. Note both equal the unconditional probabilities.

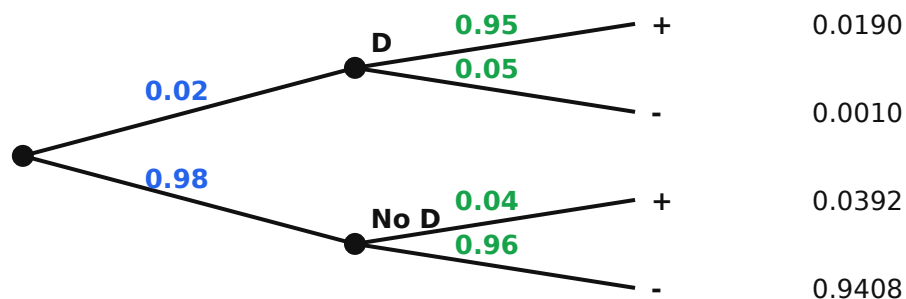
Exercise 3 - General multiplication rule

$$P(\text{1st red}) = \frac{6}{10}. \quad P(\text{2nd red} | \text{1st red}) = \frac{5}{9}.$$

$$\frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}.$$

1/3, about 0.333.

Exercise 4 - Tree diagram



Tree diagram for a medical test.

$$P(+) = 0.02(0.95) + 0.98(0.04) = 0.0190 + 0.0392 = 0.0582.$$

0.0582.

Exercise 5 - Reverse conditional

$$P(D|+) = \frac{0.0190}{0.0582} \approx 0.326.$$

About 32.6%. Most positives are false positives because the disease is rare.

Exercise 6 - Independence check

$$P(A)P(B) = 0.30 \times 0.50 = 0.15, \text{ which equals } P(A \text{ and } B).$$

Yes, A and B are independent.

Exercise 7 - Independent events

$$P(A \text{ and } B) = 0.6 \times 0.25 = 0.15. \quad P(A \text{ or } B) = 0.6 + 0.25 - 0.15 = 0.70.$$

0.15 and 0.70.

Exercise 8 - Exclusive vs independent

$$P(A \text{ and } B) = 0 \text{ but } P(A)P(B) = 0.12.$$

They are not independent: knowing A occurred means B cannot occur.

Exercise 9 - At least one

Miss all three: $0.2^3 = 0.008$. At least one: $1 - 0.008 = 0.992$.

0.992.

Exercise 10 - Table independence

Result	Tutored	Not tutored	Total
Passed	30	20	50
Failed	10	40	50
Total	40	60	100

Tutoring and passing.

$$P(\text{passed}) = \frac{50}{100} = 0.5. \quad P(\text{passed}|\text{tutored}) = \frac{30}{40} = 0.75.$$

Not independent: the tutored students were more likely to pass (0.75 compared with 0.5).

Exercise 11 - Without replacement

$$\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} = \frac{1716}{132600} = \frac{11}{850}.$$

11/850, about 0.0129.

Exercise 12 - Interpret a conditional

On rainy days 35% of buses are late; overall 12% are late.

Not independent: knowing that it rains changes the probability of being late.

Exercise 13 - Pets

Cat	Own dog	No dog	Total
Own cat	12	18	30
No cat	28	42	70
Total	40	60	100

Pet ownership.

Cat: 30 of 100, so 0.30. Dog: 40 of 100, so 0.40. Both: 12 of 100, so 0.12.

$$0.30 \times 0.40 = 0.12.$$

0.12 equals P(cat and dog), so they are independent.

Exercise 14 - Conditional with a tree

$$P(\text{defective}) = 0.6(0.02) + 0.4(0.05) = 0.012 + 0.020 = 0.032.$$

$$P(B|\text{defective}) = \frac{0.020}{0.032} = 0.625.$$

0.032 and 0.625.

Exercise 15 - Find P(B | A)

$$P(B|A) = \frac{0.18}{0.45} = 0.4.$$

P(B | A) = 0.4 = P(B), so A and B are independent.

Exercise 16 - Quality check

$$\text{Defective then good: } \frac{3}{10} \times \frac{7}{9} = \frac{21}{90}. \text{ Good then defective: } \frac{7}{10} \times \frac{3}{9} = \frac{21}{90}.$$

$$\text{Sum: } \frac{42}{90} = \frac{7}{15}.$$

7/15, about 0.467.

Exercise 17 - Two tests

$$0.9(0.8) + 0.1(0.3) = 0.72 + 0.03 = 0.75.$$

0.75.

Exercise 18 - Is it independent?

$$P(\text{junior}) = \frac{15}{30} = 0.5. \quad P(\text{junior}|\text{band}) = \frac{9}{12} = 0.75.$$

Not independent: 0.75 is not 0.5.

Exercise 19 - Multi-step

$$\text{Urgent: } 0.4(0.10) + 0.6(0.02) = 0.04 + 0.012 = 0.052.$$

$$P(\text{work}|\text{urgent}) = \frac{0.04}{0.052} \approx 0.769.$$

About 0.769.

Exercise 20 - Draw a tree

$$\text{Left-handed: } 0.4(0.30) + 0.6(0.10) = 0.12 + 0.06 = 0.18.$$

$$P(\text{guard}|\text{left}) = \frac{0.12}{0.18} \approx 0.667.$$

0.18 and about 0.667.