

AP Statistics - Confidence Intervals for a Proportion - Lesson

Common Core AP STAT 6.1, AP STAT 6.2, AP STAT 6.3, HSS-IC.B.4 · maths-pdf.fr/en/

A confidence interval for a proportion has the form $\hat{p}$ plus or minus z^* times the square root of $\hat{p}(1-\hat{p})/n$. Check the random, 10 percent, and large counts conditions, then interpret: we are C percent confident the interval captures the true proportion. The confidence level is the long-run percent of such intervals that capture the parameter.

AP Statistics · Common Core State Standards: **AP STAT 6.1, AP STAT 6.2, AP STAT 6.3, HSS-IC.B.4**

Key facts

- Interval: $\hat{p} \pm z^* \sqrt{\hat{p}(1-\hat{p})/n}$; $z^* = 1.645, 1.960, 2.576$ for 90%, 95%, 99%.
- Conditions: Random, 10% ($n < 10\%$ of population), Large counts ($n\hat{p}$ and $n(1-\hat{p})$ at least 10).
- Interpret the interval in context; interpret the level as the long-run capture rate.
- Higher confidence or smaller n gives a wider interval; quadrupling n halves the margin of error.
- A claimed value outside the interval is not plausible at that confidence level.

What is a confidence interval?

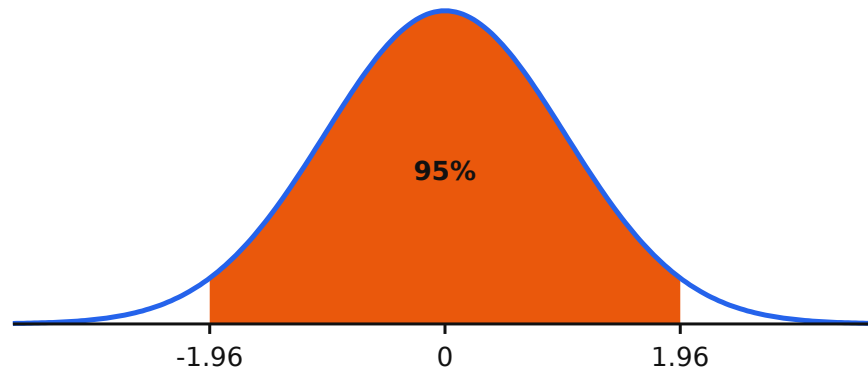
A **confidence interval** is a range of plausible values for a population parameter, built from a sample statistic. For a population proportion p , the one-sample z interval is:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

The quantity $z^* \sqrt{\hat{p}(1-\hat{p})/n}$ is the **margin of error**. The value z^* is the critical value for the chosen confidence level.

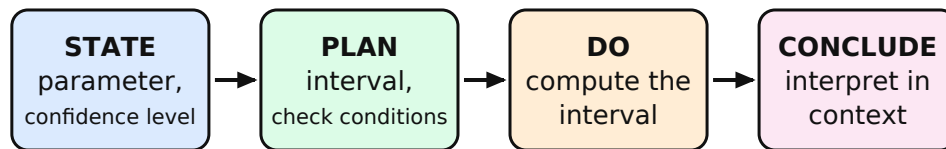
Confidence	90%	95%	99%
z^*	1.645	1.960	2.576

Common critical values.



The critical value $z^* = 1.96$ captures the middle 95% of the standard normal curve.

How do you build and check the interval?

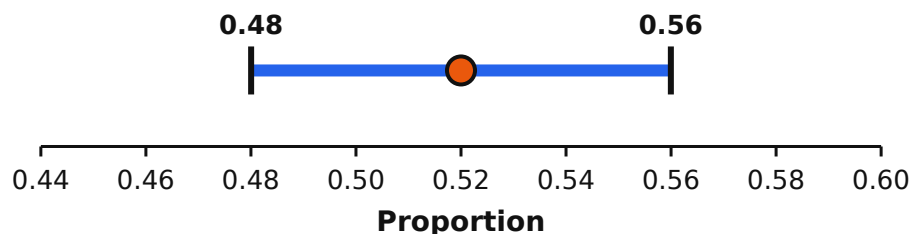


The four-step process for inference.

Example: in a random sample of $n = 600$ students, $x = 312$ ride the bus, so $\hat{p} = \frac{312}{600} = 0.52$.

- **Random:** the students were selected at random.
- **10%:** 600 is less than 10% of the school.
- **Large counts:** $n\hat{p} = 312$ and $n(1-\hat{p}) = 288$ are both at least 10.

Standard error: $\sqrt{\frac{0.52(0.48)}{600}} = 0.0204$. Margin of error: $1.96(0.0204) = 0.0400$. Interval: $(0.480, 0.560)$.



95% confidence interval for the proportion of students who use the school bus: $(0.480, 0.560)$.

The conditions matter because the formula assumes that $\hat{p}$ varies approximately normally from sample to sample. A random sample guards against bias, the 10% condition makes the sample members close to independent, and the large counts condition makes the sampling distribution of $\hat{p}$ bell-shaped. If any condition fails, the stated confidence level may not be accurate, and you should say so in your answer. Always show each condition with the numbers from the problem rather than writing only that it is satisfied.

How do you interpret the interval and the confidence level?

Interval: We are 95% confident that the interval from 0.480 to 0.560 captures the true proportion of all students who ride the bus.

Level: If we took many random samples and made a 95% interval from each, about 95% of those intervals would capture the true proportion. The confidence level describes the method, not any single interval.

$$p = 0.520.30.40.50.60.7$$

Wider intervals come from higher confidence or smaller samples. To cut the margin of error in half, multiply the sample size by 4. If a claimed value (such as 0.5) lies inside the interval, it is plausible; if it lies outside, the data provide evidence against it.

Questions about confidence intervals for a proportion

What does 95% confidence mean?

It means the method captures the true parameter in about 95% of all random samples. It is not the probability that this one fixed interval contains the parameter; the parameter is fixed and the interval is random.

What are the conditions for a z interval for a proportion?

The data must come from a random sample or random assignment, the sample must be less

than 10% of the population, and the number of successes and failures in the sample must each be at least 10.

How does sample size affect the margin of error?

The margin of error is proportional to $1/\sqrt{n}$. To halve it, you need four times as many observations.

How do I find the sample size for a desired margin of error?

Solve $n \geq (z^*/ME)^2 p^*(1 - p^*)$. If no estimate of p is available, use $p^* = 0.5$, which gives the largest, safest sample size.

Can a confidence interval be used to test a claim?

Yes. If the claimed proportion lies inside the interval, the claim is plausible; if it lies outside, the interval gives convincing evidence against the claim at that confidence level.