

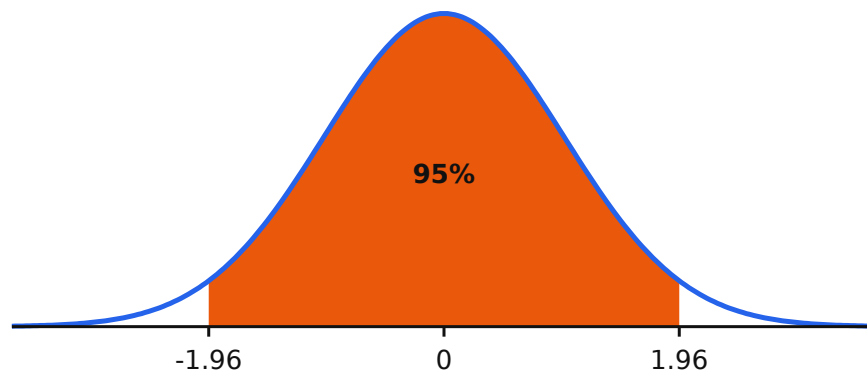
AP Statistics - Confidence Intervals for a Proportion - Answer Key

Common Core AP STAT 6.1, AP STAT 6.2, AP STAT 6.3, HSS-IC.B.4 · maths-pdf.fr/en/

Each solution lists the conditions, the standard error, the margin of error, and the interval, followed by an interpretation in context. Values are rounded to three decimals unless a sample size is requested.

AP Statistics · Common Core State Standards: **AP STAT 6.1, AP STAT 6.2, AP STAT 6.3, HSS-IC.B.4**

Exercise 1 - Critical values



The critical value $z^* = 1.96$ captures the middle 95% of the standard normal curve.

The middle C% of a standard normal curve lies between $-z^*$ and z^* .

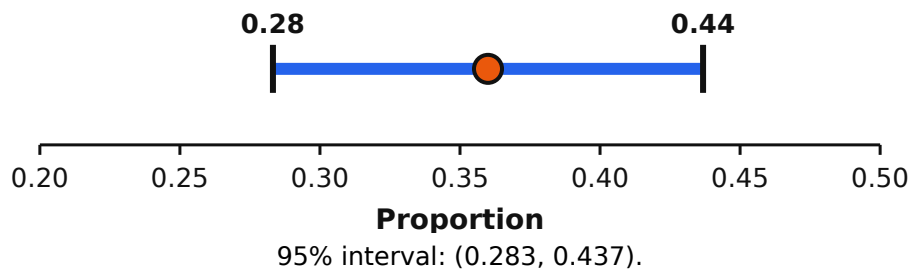
$z^* = 1.645, 1.960, 2.576$. More confidence requires capturing more area, so the interval must extend farther.

Exercise 2 - Check conditions

Random: given. 10%: $80 < 400$. Large counts: $n\hat{p} = 12$ and $n(1-\hat{p}) = 68$, both at least 10.

All conditions are met.

Exercise 3 - Compute an interval



$$\hat{p} = \frac{54}{150} = 0.36. SE = \sqrt{\frac{0.36(0.64)}{150}} = 0.0392. ME = 1.96(0.0392) = 0.0768.$$

Interval: (0.283, 0.437).

(0.283, 0.437).

Exercise 4 - Interpret the interval

Name the parameter and the population.

We are 95% confident that the interval from 0.283 to 0.437 captures the true proportion of all teens in the region who use the app daily.

Exercise 5 - Interpret the confidence level

It describes the method.

If we repeatedly took random samples of 150 teens and made a 95% interval from each, about 95% of the intervals would capture the true proportion.

Exercise 6 - Identify errors

The parameter is fixed, not random; the interval is what varies from sample to sample.

Correct wording: we are 95% confident the interval captures the true proportion, because 95% of such intervals do.

Exercise 7 - 90 percent interval

$$\hat{p} = 0.60. SE = 0.0262. ME = 1.645(0.0262) = 0.0431.$$

(0.557, 0.643).

Exercise 8 - 99 percent interval

$$ME = 2.576(0.0262) = 0.0675. \text{ Interval: } (0.533, 0.667).$$

Widths: 99%: 0.135; 90%: 0.086.

The 99% interval is wider because higher confidence requires a larger margin of error.

Exercise 9 - Margin of error

$$48\% \pm 3.1\% = (44.9\%, 51.1\%).$$

(44.9%, 51.1%). The sample proportion is typically within 3.1 percentage points of the true proportion at the stated confidence level.

Exercise 10 - Find p-hat and ME from an interval

$$\hat{p} = \frac{0.43 + 0.57}{2} = 0.50. ME = \frac{0.57 - 0.43}{2} = 0.07.$$

p-hat = 0.50; margin of error = 0.07.

Exercise 11 - Sample size

$$\text{Use } p^* = 0.5. n \geq \left(\frac{1.96}{0.04}\right)^2 (0.5)(0.5) = 600.25.$$

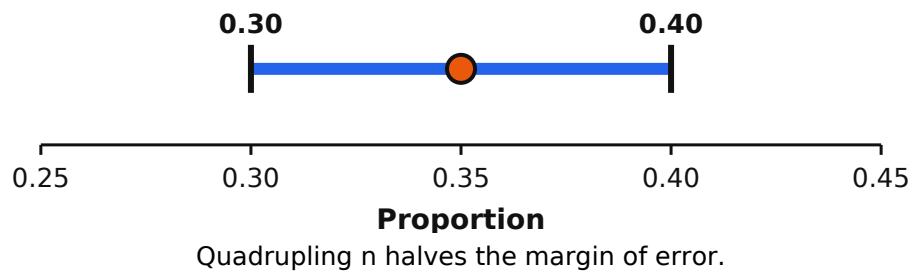
Round up: n = 601.

Exercise 12 - Sample size with an estimate

$$n \geq \left(\frac{1.96}{0.03}\right)^2 (0.3)(0.7) = 896.37.$$

n = 897.

Exercise 13 - Effect of n



Quadrupling n divides the margin of error by 2.

About 0.025.

Exercise 14 - Claim justification

The value 0.50 is above the upper bound 0.437.

No. 0.50 is not a plausible value, so the interval provides convincing evidence that the proportion is less than 0.50.

Exercise 15 - Claim not rejected

$$SE = \sqrt{\frac{0.46(0.54)}{100}} = 0.0498. \quad ME = 1.96(0.0498) = 0.0977. \quad \text{Interval: } (0.362, 0.558).$$

0.50 is inside the interval, so the claim is plausible; the data do not provide convincing evidence against it.

Exercise 16 - Large counts fails

$n\hat{p} = 3$, which is less than 10.

The large counts condition fails, so the normal model for p-hat is not appropriate.

Exercise 17 - Wider or narrower?

$$ME = z^* \sqrt{\hat{p}(1 - \hat{p})/n}.$$

(a) smaller; (b) larger; (c) larger (since $\hat{p}(1 - \hat{p})$ is largest at 0.5).

Exercise 18 - Convenience sample

The Random condition fails for a call-in poll.

No. The interval is based on a voluntary response sample, so its confidence level is meaningless; the margin of error accounts for sampling variability, not bias.

Exercise 19 - Multi-step

(a) $\hat{p} = 0.37$, $SE = 0.0216$, $ME = 0.0423$, interval (0.328, 0.412).

(b) We are 95% confident that this interval captures the true proportion of homes with a smart thermostat. (c) 0.40 lies inside the interval, so it is plausible.

(0.328, 0.412); 40% is plausible.

Exercise 20 - Two sizes

$n = 100$: $ME = 0.0960$. $n = 400$: $ME = 0.0480$.

The $n = 400$ interval is half as wide (ME 0.0480 compared with 0.0960).