

AP Statistics - t Intervals and t Tests for a Mean - Lesson

Common Core AP STAT 7.1, AP STAT 7.2, AP STAT 7.3, AP STAT 7.4, AP STAT 7.5, HSS-IC.B.4 · maths-pdf.fr/en/

When the population standard deviation is unknown, inference for a mean uses the t distribution with $n - 1$ degrees of freedom. A one-sample t interval is $\bar{x} \pm t^* s/\sqrt{n}$; a t test uses $t = (\bar{x} - \mu_0)/(s/\sqrt{n})$. Conditions: random sample, 10 percent, and a normal population or large sample. For matched pairs, analyze the differences.

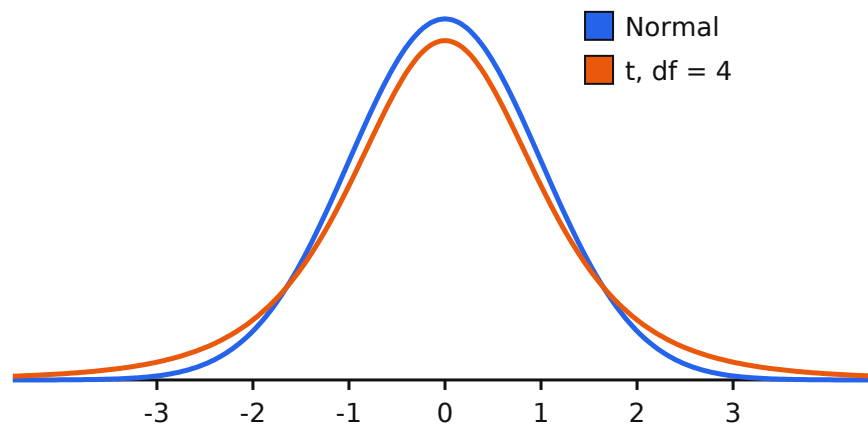
AP Statistics · Common Core State Standards: **AP STAT 7.1, AP STAT 7.2, AP STAT 7.3, AP STAT 7.4, AP STAT 7.5, HSS-IC.B.4**

Key facts

- t distribution: bell-shaped, centered at 0, heavier tails; $df = n - 1$.
- Interval: $\bar{x} \pm t^* s/\sqrt{n}$; test: $t = (\bar{x} - \mu_0)/(s/\sqrt{n})$.
- Conditions: random, 10% condition, and Normal/Large Sample (population normal, $n \geq 30$, or no strong skew or outliers).
- As df grows, t^* approaches z^* .
- Matched pairs: take differences and do a one-sample t procedure.

Why use the t distribution?

When we estimate a population mean μ and do not know the population standard deviation σ , we use the sample standard deviation s . That extra uncertainty is handled by the **t distribution**, which is bell-shaped, centered at 0, but with heavier tails than the normal. It depends on the **degrees of freedom** $df = n - 1$.



The t distribution has heavier tails than the normal; it approaches the normal as degrees of freedom grow.

The standard error of the mean is $SE_{\bar{x}} = \frac{s}{\sqrt{n}}$. As df grows, the t critical values shrink toward the normal values.

df	90%	95%	99%
5	2.015	2.571	4.032
11	1.796	2.201	3.106
20	1.725	2.086	2.845
30	1.697	2.042	2.750
z*	1.645	1.960	2.576

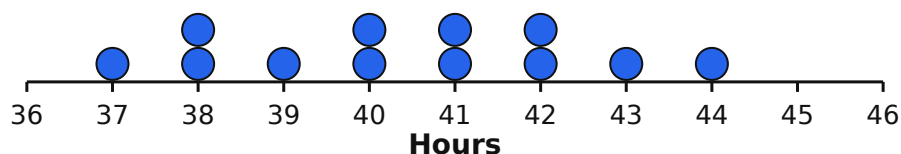
Critical values t^* decrease toward z^* as df grows.

How do you build and check a one-sample t interval?

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

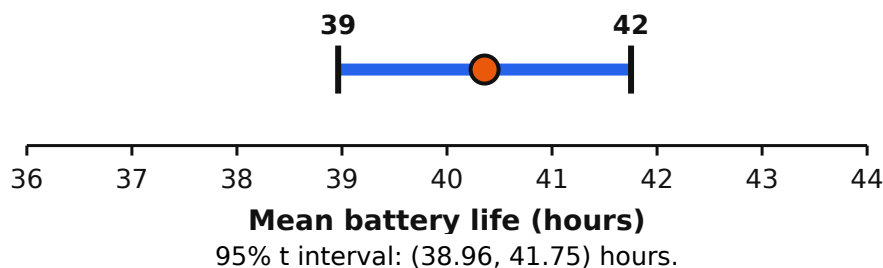
- **Random:** a random sample or randomized experiment.
- **10%:** n is less than 10% of the population when sampling without replacement.
- **Normal/Large Sample:** the population is approximately normal, or $n \geq 30$, or the sample data show no strong skew or outliers.

Example: a sample of 12 batteries has mean life 40.36 hours and $s = 2.20$ hours. The dotplot is roughly symmetric, so the Normal condition is reasonable.



Dotplot of the 12 lifetimes: roughly symmetric, no outliers, so the Normal condition is reasonable.

$SE = \frac{2.20}{\sqrt{12}} = 0.634$, $df = 11$, $t^* = 2.201$, $ME = 2.201(0.634) = 1.395$. The interval is (38.96, 41.75).



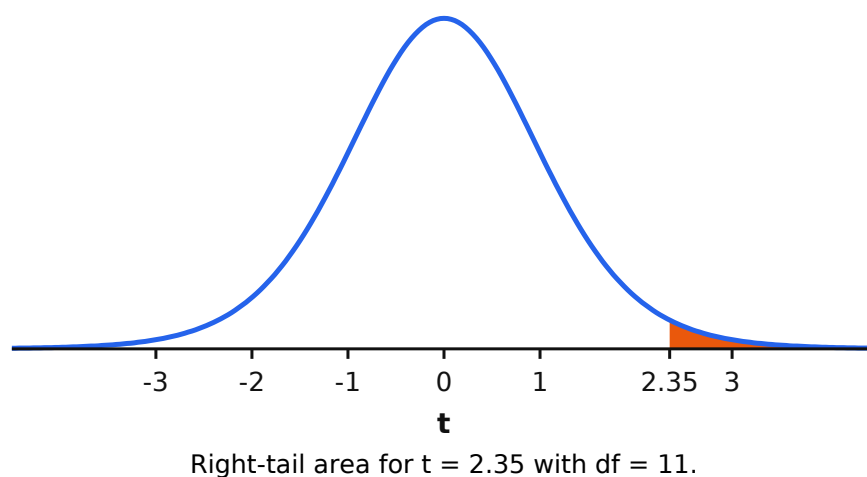
We are 95% confident that the interval from 38.96 to 41.75 hours captures the true mean battery life of all batteries of this type.

How do you carry out a one-sample t test?

For $H_0: \mu = \mu_0$, the test statistic is:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}, \quad df = n - 1$$

The p-value is the area in the tail(s) of the t distribution with df degrees of freedom. Example: if $t = 2.35$ with $df = 11$ and $H_a: \mu > \mu_0$, the p-value is 0.0192.



Because 0.0192 is less than 0.05, we reject H_0 and conclude that there is convincing evidence that the mean is greater than μ_0 .

What about matched pairs?

When each individual is measured twice (before and after), compute the **differences** and treat them as one sample. Use a one-sample t procedure on the differences, with $H_0: \mu_d = 0$. Check Normality using the differences, not the original data.

Questions about t intervals and t tests for a mean

Why is the t distribution used instead of the normal?

Using s instead of sigma adds variability, so the standardized statistic has heavier tails than the normal. The t distribution accounts for that, especially for small samples.

What are degrees of freedom?

For a one-sample t procedure, $df = n - 1$. They tell which t curve to use; larger df gives a curve closer to the normal curve.

How do I check the Normal condition for a small sample?

Plot the data. If the graph shows no strong skewness and no outliers, a t procedure is reasonable. For very small samples, you need a population that is approximately normal.

When do I use a matched pairs t procedure?

When each subject is measured twice or two similar subjects are paired. Compute the differences and run a one-sample t procedure on them.

Can I use a t procedure with a skewed population?

If the sample is large (n at least 30), the central limit theorem makes the sampling distribution of x-bar approximately normal, so yes. With small n and clear skew or outliers, avoid t procedures.