

AP Statistics - z-Scores and the Normal Distribution - Lesson

Common Core AP STAT 1.10, AP STAT 1.11, HSS-ID.A.4 · maths-pdf.fr/en/

A **z-score** gives the number of standard deviations a value is from the mean: $z = (x - \text{mean})/\text{sd}$. A normal distribution is a bell-shaped curve defined by its mean and standard deviation, and areas under it are proportions of data. The empirical rule says about 68%, 95%, and 99.7% of values lie within 1, 2, and 3 standard deviations.

AP Statistics · Common Core State Standards: **AP STAT 1.10, AP STAT 1.11, HSS-ID.A.4**

Key facts

- $z = (x - \text{mean}) / \text{sd}$ measures position in standard deviations.
- Empirical rule: 68% within 1 sd, 95% within 2 sd, 99.7% within 3 sd.
- Area under the curve over an interval is the proportion of values in it; total area is 1.
- Use normalcdf for areas and invNorm for percentiles; always sketch and shade first.
- A percentile is the percent of values at or below a given value.

What is a z-score?

A **z-score** tells how many standard deviations a value lies from the mean:

$$z = \frac{x - \mu}{\sigma}$$

A positive z is above the mean and a negative z is below it. A score of 130 on a test with mean 100 and standard deviation 15 has $z = \frac{130 - 100}{15} = 2$. z -scores let you compare values from different distributions.

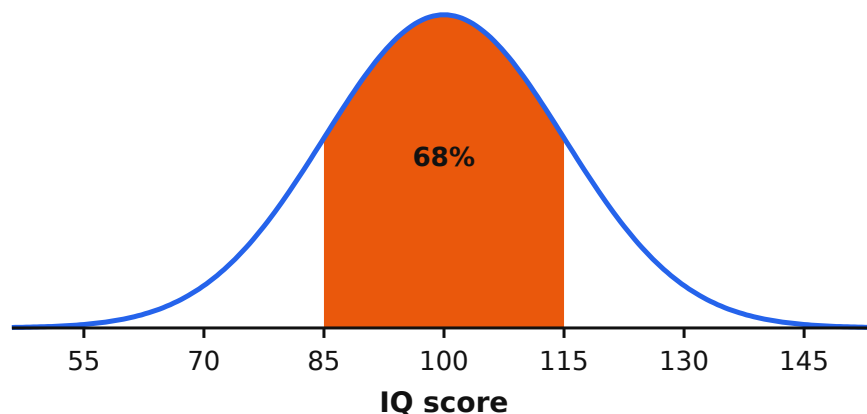
What is the normal distribution?

A **normal distribution** is a symmetric, bell-shaped density curve described by its mean μ and standard deviation σ . The total area under the curve is 1, and the area over an interval is the proportion of values in that interval.

Empirical rule (68-95-99.7)

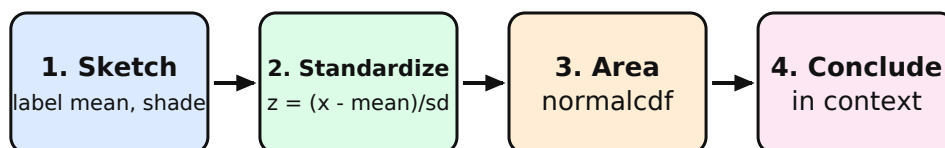
In a normal distribution, about 68% of the values are within 1 standard deviation of the

mean, about 95% within 2, and about 99.7% within 3.



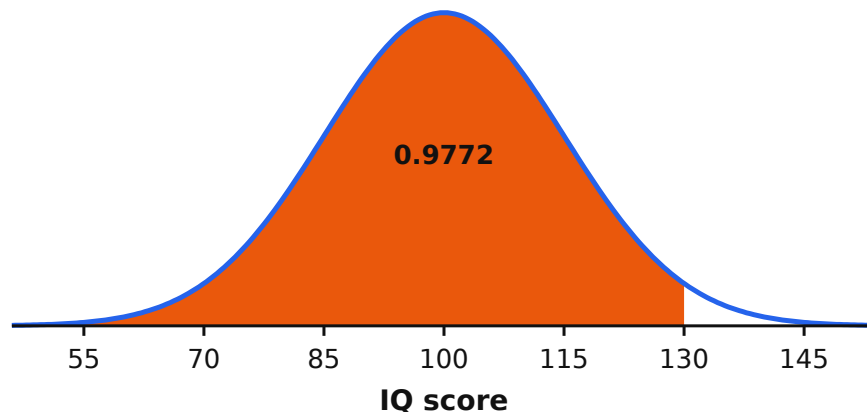
The empirical rule: about 68% of values lie within 1 standard deviation of the mean, 95% within 2, and 99.7% within 3.

How do you find a normal probability?



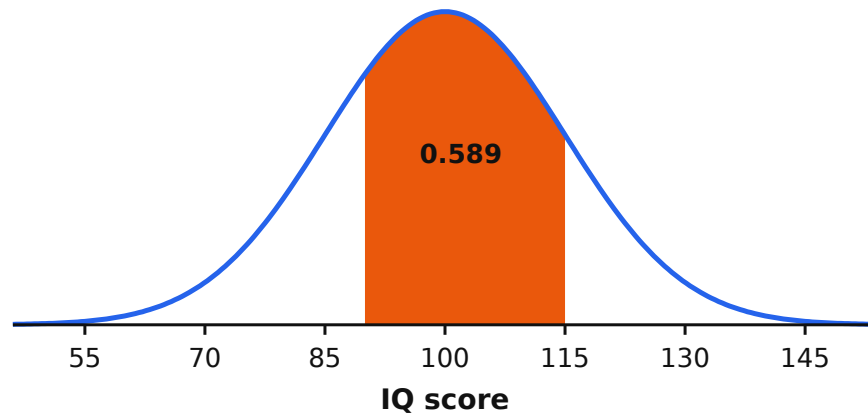
Draw first: a shaded sketch prevents most errors.

1. Sketch a normal curve, label the mean, and shade the region.
2. Convert the boundary to a z-score.
3. Use a table or the calculator command `normalcdf(lower, upper, mean, sd)` to get the area.
4. Answer in context.



A score of 130 has $z = 2$. Its percentile is the shaded area to the left.

Because $P(Z < 2) \approx 0.9772$, a score of 130 is at about the 97.7th percentile. The percentile of a value is the percent of values at or below it.



The area between 90 and 115 is about 0.589.

For the area between 90 and 115, use $z = \frac{90-100}{15} = -0.67$ and $z = \frac{115-100}{15} = 1$. The area is $0.8413 - 0.2525 = 0.5888$, so about 58.9% of scores fall between 90 and 115.

How do you work backward from a percentile?

To find the value at a given percentile, find the z-score with that area to its left (invNorm on the calculator), then compute $x = \mu + z\sigma$. The 90th percentile has $z \approx 1.2816$, so on this scale it is $100 + 1.2816(15) \approx 119.2$.

Questions about z-scores and the normal distribution

What does a z-score of -1.5 mean?

It means the value is 1.5 standard deviations below the mean. The sign shows the direction and the size shows the distance in units of standard deviation.

How do I compare scores from different tests?

Convert each score to a z-score. The larger z-score is relatively better because it is farther above its own mean in standard deviation units.

When can I use the empirical rule?

Use it only when the distribution is approximately normal, meaning unimodal, symmetric, and bell-shaped. For other shapes, the 68-95-99.7 percents may not hold.

What is the difference between normalcdf and invNorm?

Normalcdf takes boundaries and returns an area (a probability or proportion). InvNorm takes an area to the left and returns the boundary value or z-score.

Is the probability of an exact value in a normal distribution zero?

Yes. A continuous distribution assigns area to intervals, not to single points, so $P(X = 5) = 0$ and $P(X < 5)$ equals $P(X \leq 5)$.