

AP Statistics - z-Scores and the Normal Distribution - Answer Key

Common Core AP STAT 1.10, AP STAT 1.11, HSS-ID.A.4 · maths-pdf.fr/en/

Each solution shows the z-score, the area from the standard normal table or normalcdf, and a sentence in context. Values are rounded to four decimals for probabilities and to two decimals for z-scores.

AP Statistics · Common Core State Standards: **AP STAT 1.10, AP STAT 1.11, HSS-ID.A.4**

Exercise 1 - Compute a z-score

$$z = \frac{69-64}{2.5} = 2.00.$$

z = 2.00; she is 2 standard deviations above the mean.

Exercise 2 - Value from a z-score

$$x = \mu + z\sigma = 78 + (-1.5)(6) = 69.$$

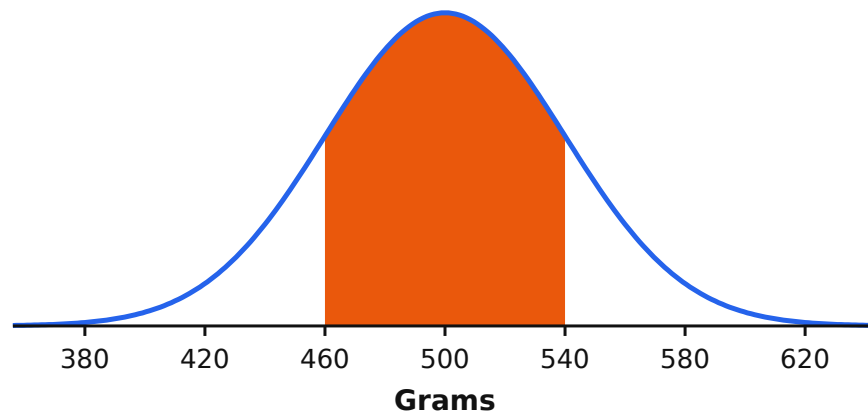
69.

Exercise 3 - Compare two scores

$$\text{Jamal: } z = \frac{84-75}{6} = 1.5. \text{ Sofia: } z = \frac{640-600}{50} = 0.8.$$

Jamal did relatively better (z = 1.5 compared with 0.8).

Exercise 4 - Empirical rule



Weights of a bag of rice in grams.

420 and 580 are 2 standard deviations from 500, so about 95%.

460 and 540 are 1 standard deviation away, so about 68%.

About 95%; about 68%.

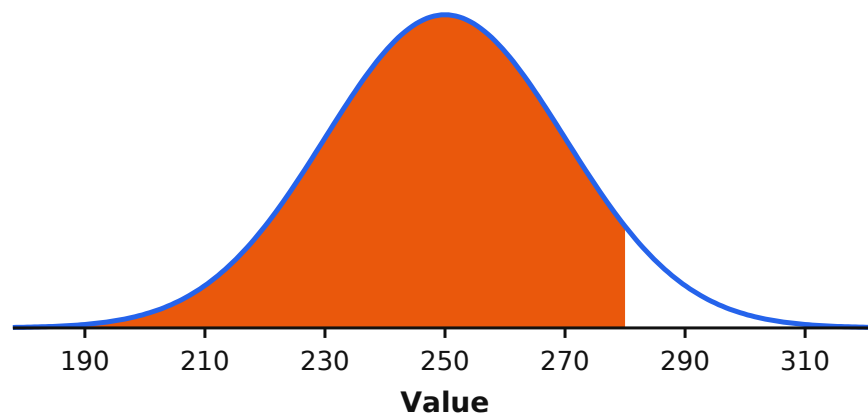
Exercise 5 - Empirical rule counts

15 is 2 standard deviations above the mean. About 95% are within 2, so 5% are outside, and half of those, 2.5%, are above.

$$0.025 \times 400 = 10.$$

About 10 batteries.

Exercise 6 - Area to the left



Area left of 280.

$z = \frac{280-250}{20} = 1.5$. From the table or calculator, $P(Z < 1.5) = 0.9332$.

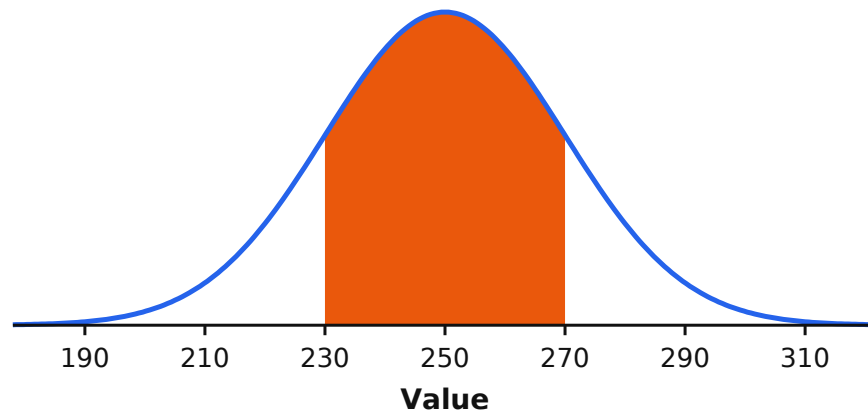
0.9332, about 93.3%.

Exercise 7 - Area to the right

$z = \frac{215-250}{20} = -1.75$. $P(Z > -1.75) = 1 - 0.0401 = 0.9599$.

0.9599.

Exercise 8 - Area between



Area between 230 and 270.

The boundaries have $z = -1$ and $z = 1$. $0.8413 - 0.1587 = 0.6827$.

0.6827, matching the 68% of the empirical rule.

Exercise 9 - Value from a percentile

For area 0.90 to the left, $z = 1.28$. $x = 100 + (1.28)(15) = 119.2$.

About 119.2.

Exercise 10 - Percentile of a value

$z = \frac{22-27}{4} = -1.25$. $P(Z < -1.25) = 0.1056$.

About the 10.6th percentile: about 10.6% of runners were faster than 22 minutes. (A

lower time is faster, so 10.6% of runners were at or below her time.)

Exercise 11 - Quartiles of a normal

$$z_{Q_1} = -0.6745, z_{Q_3} = 0.6745.$$

$$Q_1 = 100 - 0.6745(15) = 89.9, Q_3 = 110.1.$$

$$Q1 = 89.9, Q3 = 110.1, IQR = 20.2.$$

Exercise 12 - Change of units

In centimeters: mean 162.56, sd 6.35, height 175.26. $z = \frac{175.26 - 162.56}{6.35} = 2.00$.

$z = 2$; z-scores do not change under a change of units.

Exercise 13 - Is it approximately normal?

Within 1 sd: $\frac{138}{200} = 69\%$. Within 2 sd: $\frac{192}{200} = 96\%$.

These are close to 68% and 95%, so the data are approximately normal (a histogram should also be checked).

Exercise 14 - Bottle filling

$$z = \frac{11.8 - 12}{0.15} = -1.33. P(Z < -1.33) = 0.0918.$$

$$0.0918 \times 5000 = 459.$$

About 9.18%; about 459 bottles.

Exercise 15 - Tall women

$$z = \frac{70 - 64}{2.5} = 2.4. P(Z > 2.4) = 1 - 0.9918 = 0.0082.$$

About 0.82%. It is unusual ($z = 2.4$ is beyond 2 standard deviations).

Exercise 16 - Commute times

$$z = \frac{55-60}{4} = -1.25 \text{ and } z = 1.25. 0.8944 - 0.1056 = 0.7887.$$

About 78.9%.

Exercise 17 - Top 5 percent

The 95th percentile has $z = 1.645$.

$$x = 100 + 1.645(15) = 124.7.$$

About 124.7, so an IQ of at least 125 (rounded up).

Exercise 18 - Find the standard deviation

2.5% above means 4.0 is 2 standard deviations above the mean.

$$2\sigma = 4.0 - 3.2 = 0.8, \text{ so } \sigma = 0.4.$$

About 0.4 oz.

Exercise 19 - Two-step

Within 0.5: $z = \pm 0.625$, $P = 0.4680$.

More than 1 inch above: $z = 1.25$, $P = 0.1056$.

About 46.8% and 10.6%.

Exercise 20 - Scores above a cutoff

$$z = \frac{90-72}{9} = 2. P(Z > 2) = 1 - 0.9772 = 0.0228.$$

About 2.3%.