

AP Statistics - Probability Rules and Simulation - Lesson

Common Core AP STAT 4.1, AP STAT 4.2, AP STAT 4.3, AP STAT 4.4, HSS-CP.A.1, HSS-CP.B.7 · maths-pdf.fr/en/

Probability is the long-run relative frequency of an outcome, a number from 0 to 1, and the law of large numbers says relative frequencies settle down as trials increase. Use simulation to estimate hard probabilities, the complement rule $P(\text{not } A) = 1 - P(A)$, and the addition rule $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

AP Statistics · Common Core State Standards: **AP STAT 4.1, AP STAT 4.2, AP STAT 4.3, AP STAT 4.4, HSS-CP.A.1, HSS-CP.B.7**

Key facts

- Probability = long-run relative frequency; always between 0 and 1.
- Law of large numbers: more trials bring the relative frequency closer to the probability.
- Complement rule: $P(A \text{ not happening}) = 1 - P(A)$.
- General addition rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$; subtract nothing if A and B are mutually exclusive.
- Simulation: assign digits, simulate trials, repeat many times, then estimate.

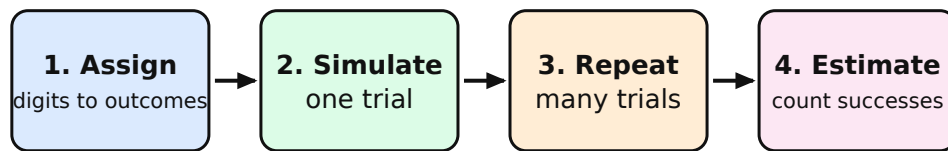
What does probability mean?

The **probability** of an outcome is its long-run relative frequency, a number between 0 and 1. The **law of large numbers** says that as a random process is repeated more times, the proportion of times an event occurs gets closer to its true probability. The graph below shows coin flips settling near 0.5.

050100150200Number of coin flips0.000.250.500.751.00Proportion of heads

Random does not mean evenly spaced: short runs can look streaky. A coin that landed heads four times in a row is still 50% likely to land heads next.

How do you estimate probabilities with a simulation?



Four steps for a simulation.

To simulate a free-throw shooter who makes 70% of her shots, let the digits 0 to 6 mean "made" and 7 to 9 mean "missed." Read one digit per shot, count successes in each trial, and repeat many times.

What are the basic probability rules?

- **Sample space** and equally likely outcomes: $P(A) = \frac{\text{number of outcomes in } A}{\text{total outcomes}}$.
- **Complement rule:** $P(A^C) = 1 - P(A)$.
- **Mutually exclusive** events cannot happen together: $P(A \text{ or } B) = P(A) + P(B)$.
- **General addition rule:** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

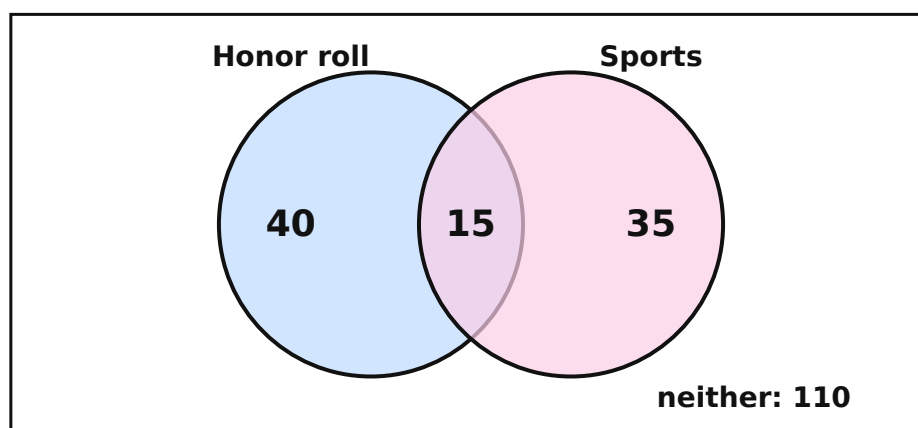
Sample space for two dice: 36 equally likely outcomes.

For two dice, 6 of the 36 outcomes have a sum of 7, so $P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$.

When should you use the complement?

Phrases such as "at least one" are easiest through the complement. The only way to get no 6 in four rolls of a die is to roll a non-6 every time, with probability $\left(\frac{5}{6}\right)^4 \approx 0.482$. So the probability of at least one 6 in four rolls is $1 - 0.482 = 0.518$. Computing "at least one" directly would require adding many separate cases, which is slow and error-prone. Whenever a question asks for the chance of something happening at least once, start by asking what the opposite event looks like.

How do Venn diagrams help?



Venn diagram of 200 students. $P(\text{honor roll or sports}) = (40 + 15 + 35)/200$.

Out of 200 students, 55 are on the honor roll (40+15) and 50 play sports (15+35). The general addition rule gives $P(H \text{ or } S) = \frac{55}{200} + \frac{50}{200} - \frac{15}{200} = \frac{90}{200} = 0.45$. Equivalently, the complement gives $1 - \frac{110}{200} = 0.45$.

Questions about probability rules and simulation

What is the law of large numbers?

It states that the relative frequency of an event in many repetitions gets closer to the true probability. It does not say that a streak will be 'made up for' in the next few trials.

What does mutually exclusive mean?

Two events are mutually exclusive if they cannot occur at the same time, so $P(A \text{ and } B) = 0$. A single die roll cannot be both a 2 and a 5.

What is the complement of an event?

The complement is the event that A does not happen. Its probability is $1 - P(A)$, which is useful for 'at least one' problems.

How many trials should a simulation use?

More is better, because of the law of large numbers. A handful of trials can be far off, while hundreds or thousands give a stable estimate.

Is a probability of 0.5 the same as 'it will alternate'?

No. Independent trials do not remember the past. A fair coin has a 0.5 chance of heads on every flip, no matter how many tails came first.