

AP Statistics - Probability Rules and Simulation - Answer Key

Common Core AP STAT 4.1, AP STAT 4.2, AP STAT 4.3, AP STAT 4.4, HSS-CP.A.1, HSS-CP.B.7 · maths-pdf.fr/en/

Each solution shows the sample space or the rule used and computes every probability exactly, with decimals rounded to three places when needed. Simulation results use the digits given in the problem.

AP Statistics · Common Core State Standards: **AP STAT 4.1, AP STAT 4.2, AP STAT 4.3, AP STAT 4.4, HSS-CP.A.1, HSS-CP.B.7**

Exercise 1 - Interpret a probability

Probability describes long-run frequency.

In many days with conditions like tomorrow's, it rains on about 30% of them.

Exercise 2 - Law of large numbers

With few flips, chance variation is large; with many flips, the proportion is close to the true probability 0.5.

The law of large numbers: the relative frequency approaches the true probability as flips increase.

Exercise 3 - Gambler's fallacy

Coin flips are independent; the coin has no memory.

No: P(heads) is still 0.5. The long-run proportion balances out through many more flips, not by compensation.

Exercise 4 - Equally likely outcomes

Total 10. $P(\text{red}) = \frac{5}{10} = 0.5$. $P(\text{blue or green}) = \frac{3+2}{10} = 0.5$.

0.5 and 0.5.

Exercise 5 - Sample space of two coins

HH, HT, TH, TT are equally likely. Exactly one head: HT, TH.

$$P = \frac{2}{4} = 0.5.$$

0.5.

Exercise 6 - Two dice

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Sample space for two dice: 36 equally likely outcomes.

Sum 7: 6 outcomes. Sum 11: (5,6), (6,5), 2 outcomes.

$$\frac{6}{36} = \frac{1}{6} \text{ and } \frac{2}{36} = \frac{1}{18}.$$

1/6 and 1/18.

Exercise 7 - Complement rule

$$1 - 0.82 = 0.18.$$

0.18.

Exercise 8 - At least one

No 6 on either die: $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$. Complement: $1 - \frac{25}{36} = \frac{11}{36}$.

11/36, about 0.306.

Exercise 9 - Mutually exclusive events

A card cannot be both a king and a queen.

$$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}.$$

2/13, about 0.154.

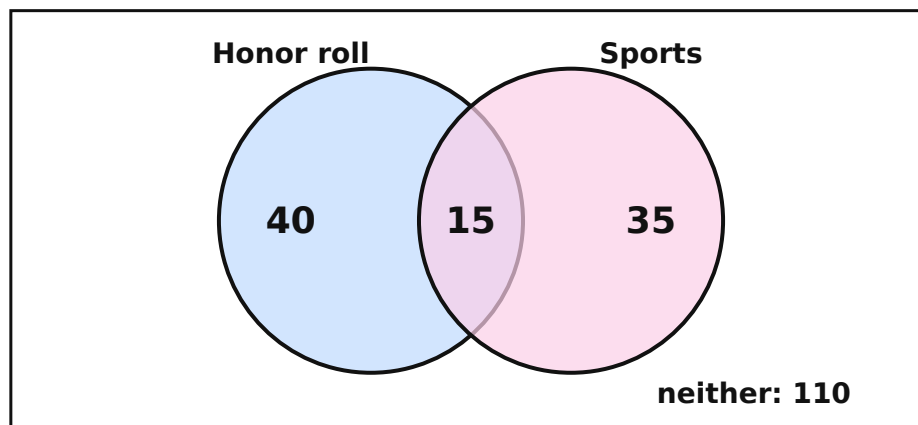
Exercise 10 - General addition rule

$$P(\text{heart}) = \frac{13}{52}, P(\text{king}) = \frac{4}{52}, P(\text{king of hearts}) = \frac{1}{52}.$$

$$\frac{13+4-1}{52} = \frac{16}{52} = \frac{4}{13}.$$

4/13, about 0.308.

Exercise 11 - Read a Venn diagram



Venn diagram of 200 students. $P(\text{honor roll or sports}) = (40 + 15 + 35)/200$.

$$\text{Honor roll: } \frac{40+15}{200} = 0.275. \text{ Sports: } \frac{15+35}{200} = 0.25. \text{ Both: } \frac{15}{200} = 0.075.$$

0.275, 0.25, and 0.075.

Exercise 12 - Neither

$$\text{Or: } 0.275 + 0.25 - 0.075 = 0.45. \text{ Neither: } 1 - 0.45 = 0.55, \text{ which matches } \frac{110}{200}.$$

0.45 and 0.55.

Exercise 13 - Are they mutually exclusive?

$$P(A \text{ and } B) = 0.5 + 0.3 - 0.6 = 0.2.$$

$P(A \text{ and } B) = 0.2$, which is not 0, so A and B are not mutually exclusive.

Exercise 14 - Run a simulation

Random digits
0 3 8 1 5 6 4 9 2 7 6 1 0 5 8 2 3 4 9 6 7 1 5 2 8 3 0 6 4 9

Thirty random digits for the simulation.

Trials: 0 3 8 -> 2; 1 5 6 -> 3; 4 9 2 -> 2; 7 6 1 -> 2; 0 5 8 -> 2; 2 3 4 -> 3; 9 6 7 -> 1; 1 5 2 -> 3; 8 3 0 -> 2; 6 4 9 -> 2 made shots.

Trials with at least 2 made: 9 of 10.

Estimated probability 0.9 (more trials give a better estimate).

Exercise 15 - Compare with the exact value

$$\text{Exact: } 3(0.7)^2(0.3) + (0.7)^3 = 0.441 + 0.343 = 0.784.$$

The estimate (0.9) differs from 0.784 because only 10 trials were run. A simulation with hundreds of trials would be closer.

Exercise 16 - Design a simulation

Use a random digit generator. Digits 1 to 25 are not available as single digits, so use two-digit pairs 00 to 99: 00 to 24 means prize, 25 to 99 means no prize.

Read pairs until the first prize, record how many boxes, and repeat many times; average the counts.

Assign 00-24 as prize, read pairs until a prize, record the count, repeat many trials and average.

Exercise 17 - Probability of a union

$P(A \text{ or } B) = 0.4 + 0.35 = 0.75$. Neither: $1 - 0.75 = 0.25$.

0.75 and 0.25.

Exercise 18 - Raffle

First prize: $\frac{8}{200} = 0.04$.

Win none in 3 draws: $\frac{192}{200} \times \frac{191}{199} \times \frac{190}{198} \approx 0.8842$.

0.04; about 0.884 chance of winning nothing.

Exercise 19 - Multi-step

$0.60 + 0.45 - 0.25 = 0.80$. Neither: $1 - 0.80 = 0.20$.

80% have at least one; 20% have neither.

Exercise 20 - At least one six

No 6 in a roll has probability $5/6$, and the rolls are independent: $\left(\frac{5}{6}\right)^4 = \frac{625}{1296} \approx 0.482$.

Complement: $1 - 0.482 = 0.518$.

About 0.518.