

AP Statistics - Random Variables, Expected Value and Combining Variables - Answer Key

Common Core AP STAT 4.7, AP STAT 4.8, AP STAT 4.9, HSS-MD.A.1, HSS-MD.A.2, HSS-MD.B.5 · maths-pdf.fr/en/

Each solution shows the products x times p , the squared deviations, and the combination rule used. All standard deviations of combinations assume independence, which is stated in the problem.

AP Statistics · Common Core State Standards: **AP STAT 4.7, AP STAT 4.8, AP STAT 4.9, HSS-MD.A.1, HSS-MD.A.2, HSS-MD.B.5**

Exercise 1 - Valid distribution?

The probabilities must be between 0 and 1 and add to 1.

(a) Sum 1.1, no. (b) Sum 1.0, yes. (c) Contains -0.1, no.

Only (b) is valid.

Exercise 2 - Missing probability

$$P(2) = 1 - 0.1 - 0.4 - 0.2 = 0.3.$$

0.3.

Exercise 3 - Find a probability

$$P(X \geq 3) = 0.20 + 0.15 + 0.10 = 0.45. \quad P(X < 3) = 0.30 + 0.25 = 0.55.$$

0.45 and 0.55.

Exercise 4 - Expected value

x	p	x p	(x - 2.5)^2 p
1	0.3	0.30	0.6750
2	0.25	0.50	0.0625
3	0.2	0.60	0.0500
4	0.15	0.60	0.3375
5	0.1	0.50	0.6250
Sum	1	2.50	1.7500

Mean 2.50; variance 1.7500; standard deviation 1.323.

$$1(0.30)+2(0.25)+3(0.20)+4(0.15)+5(0.10)=0.30+0.50+0.60+0.60+0.50=2.5.$$

2.5 items.

Exercise 5 - Interpret the mean

The expected value is a long-run average.

Over many plays, the player loses an average of 50 cents per play.

Exercise 6 - Standard deviation

x	p	x p	(x - 2.5)^2 p
1	0.3	0.30	0.6750
2	0.25	0.50	0.0625
3	0.2	0.60	0.0500
4	0.15	0.60	0.3375
5	0.1	0.50	0.6250
Sum	1	2.50	1.7500

Mean 2.50; variance 1.7500; standard deviation 1.323.

$$\text{Variance: } 1.7500. \sigma_X = \sqrt{1.7500} = 1.323.$$

1.323 items: orders typically differ from the mean of 2.5 items by about 1.4 items.

Exercise 7 - Expected winnings

$$4(0.2) + (-1)(0.8) = 0.8 - 0.8 = 0.$$

\$0, so the game is fair.

Exercise 8 - Raffle

Net win: $500 - 5 = 495$ with probability 0.001; net loss \$5 with probability 0.999.

$$495(0.001) + (-5)(0.999) = 0.495 - 4.995 = -4.50.$$

-\$4.50 per ticket.

Exercise 9 - Linear transformation

$$\mu_Y = 3 + 2(2.5) = 8, \sigma_Y = 2(1.323) = 2.646.$$

Mean \$8; standard deviation \$2.65.

Exercise 10 - Sum of means

$$\text{Mean: } 40 + 25 = 65. \sigma = \sqrt{3^2 + 4^2} = \sqrt{25} = 5.$$

Mean 65 minutes; standard deviation 5 minutes.

Exercise 11 - Difference

$$\text{Mean: } 40 - 25 = 15. \text{ Standard deviation: } \sqrt{3^2 + 4^2} = 5.$$

Mean 15 minutes; standard deviation 5 minutes (same as the sum).

Exercise 12 - Standard deviations do not add

For independent variables, variances add, not standard deviations.

$$\sigma = \sqrt{9 + 16} = 5, \text{ not } 7.$$

Exercise 13 - 2X versus X + X

$$2X: 2(6)=12. \quad X + Y: \sqrt{6^2+6^2}=\sqrt{72}\approx 8.49.$$

12 compared with about 8.49: doubling one copy varies more than adding two independent copies.

Exercise 14 - Insurance

Outcome	Profit	Probability
No claim	\$400	0.95
\$3,000 claim	-\$2,600	0.04
\$20,000 claim	-\$19,600	0.01

Profit for the insurer.

$$\text{Profit: } 400, 400-3000=-2600, 400-20000=-19600.$$

$$400(0.95)+(-2600)(0.04)+(-19600)(0.01)=380-104-196=80.$$

\$80 per customer per year.

Exercise 15 - Total of two games

$$2+1=3. \quad \sqrt{9+16}=5.$$

Mean \$3; standard deviation \$5.

Exercise 16 - Weights

$$\text{Mean: } 10\times 12=120. \quad \text{Variance: } 10\times 0.25=2.5. \quad \sigma=\sqrt{2.5}\approx 1.58.$$

Mean 120 oz; standard deviation about 1.58 oz.

Exercise 17 - Normal approximation

$$z=\frac{10-8}{1.414}=1.41. \quad P(Z>1.41)=0.0793.$$

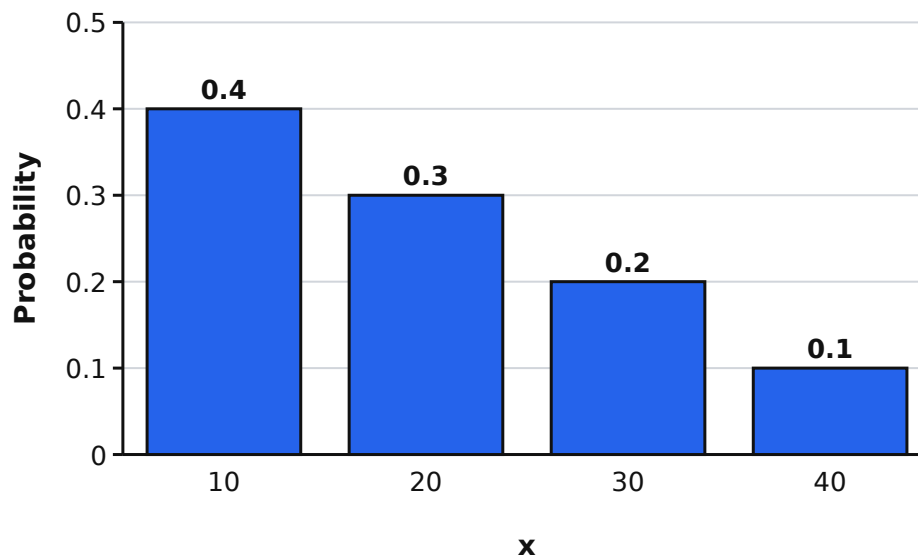
About 0.0793.

Exercise 18 - Change of units

$$1.8(20) + 32 = 68. \quad 1.8(4) = 7.2.$$

Mean 68°F; standard deviation 7.2°F.

Exercise 19 - Complete a distribution



Completed distribution.

$$P(40) = 1 - 0.9 = 0.1. \quad \text{Mean: } 10(0.4) + 20(0.3) + 30(0.2) + 40(0.1) = 4 + 6 + 6 + 4 = 20.$$

P(40) = 0.1; mean 20.

Exercise 20 - Multi-step

$$E(X) = 0(0.5) + 1(0.3) + 2(0.2) = 0.7. \quad \text{Profit: } 1.50X - 0.40, \text{ so } 1.50(0.7) - 0.40 = 0.65.$$

\$0.65 per minute.