

AP Statistics - Sampling Distributions and the Central Limit Theorem - Answer Key

Common Core AP STAT 5.1, AP STAT 5.2, AP STAT 5.3, AP STAT 5.4, AP STAT 5.5, AP STAT 5.6, AP STAT 5.7, HSS-IC.A.1, HSS-IC.A.2 · maths-pdf.fr/en/

Each solution lists the mean, the standard deviation, and the conditions, then standardizes and finds the area with the normal table or normalcdf. Probabilities are rounded to four decimals.

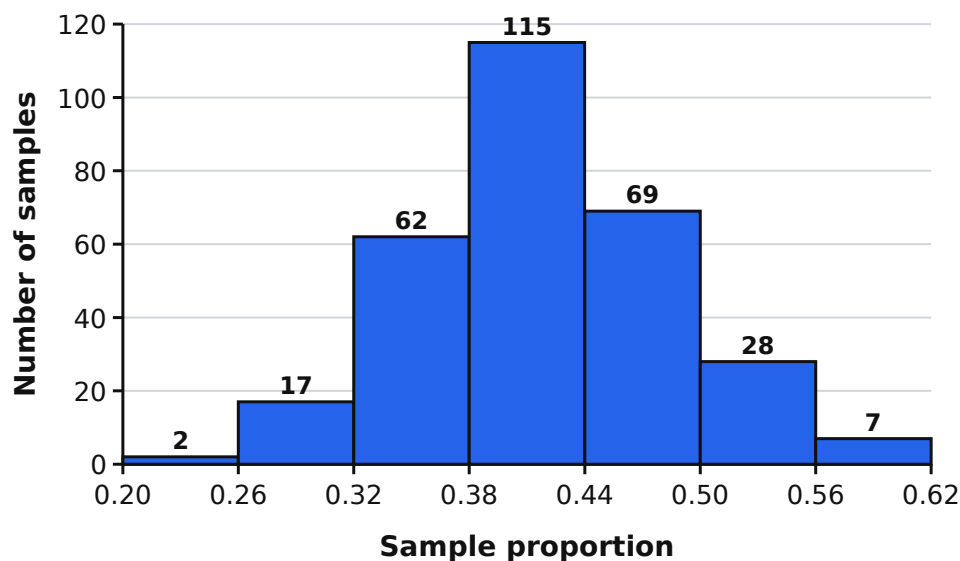
AP Statistics · Common Core State Standards: **AP STAT 5.1, AP STAT 5.2, AP STAT 5.3, AP STAT 5.4, AP STAT 5.5, AP STAT 5.6, AP STAT 5.7, HSS-IC.A.1, HSS-IC.A.2**

Exercise 1 - Parameter or statistic

The parameter describes the population; the statistic describes the sample.

Parameter: $p = 0.56$; statistic: $\hat{p} = 0.53$.

Exercise 2 - Sampling variability



300 simulated sample proportions: centered at 0.40, approximately normal.

The distribution is roughly normal and centered near 0.40, with most values between about 0.26 and 0.54.

Approximately normal, centered at 0.40, spread about 0.07. A single sample

proportion will usually fall within about 0.14 of p.

Exercise 3 - Mean and SD of p-hat

$$\mu_{\hat{p}} = 0.30, \sigma_{\hat{p}} = \sqrt{\frac{0.30(0.70)}{150}} = 0.0374.$$

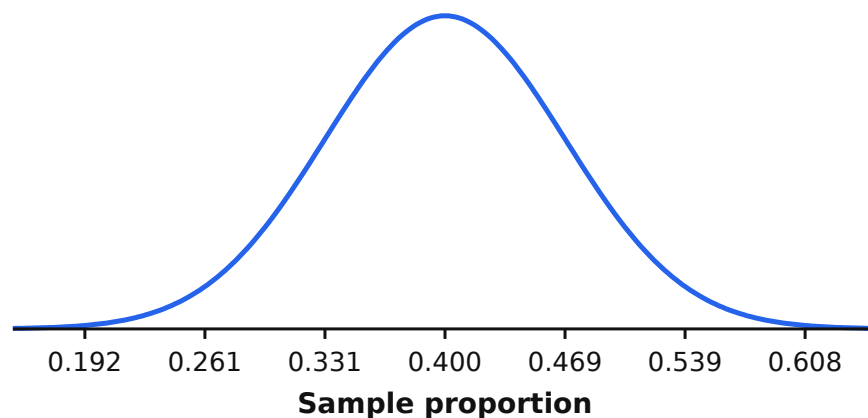
Mean 0.30; standard deviation 0.0374.

Exercise 4 - Check conditions

$$np = 60(0.08) = 4.8 \text{ is less than } 10.$$

No: $np = 4.8 < 10$, so the sampling distribution is not approximately normal (the 10% condition holds since $60 < 90$).

Exercise 5 - Probability for p-hat



Sampling distribution of p-hat for $n = 50$: mean 0.40, standard deviation 0.069.

$$z = \frac{0.50 - 0.40}{0.0693} = 1.44. P(Z > 1.44) = 0.0749.$$

About 0.0749.

Exercise 6 - Effect of sample size

$n = 50$: 0.0693. $n = 200$: 0.0346. The ratio is 2.00.

Quadrupling n cuts the standard deviation in half.

Exercise 7 - Unbiased estimator

The center of the sampling distribution should equal the parameter.

Biased (it overestimates by 2). A larger sample reduces variability but does not remove the bias.

Exercise 8 - Mean and SD of x-bar

$$\mu_{\bar{x}} = 1200, \sigma_{\bar{x}} = \frac{300}{\sqrt{36}} = 50.$$

Mean \$1,200; standard deviation \$50.

Exercise 9 - Probability for x-bar

$$\sigma_{\bar{x}} = \frac{3}{\sqrt{9}} = 1, z = \frac{71-70}{1} = 1, P(Z > 1) = 0.1587.$$

0.1587.

Exercise 10 - Individual versus mean

$$z = \frac{71-70}{3} = 0.33, P(Z > 0.33) = 0.3707.$$

0.3707, much larger than 0.1587; sample means vary less than individuals.

Exercise 11 - Central limit theorem

$$\mu = 5, \sigma_{\bar{x}} = \frac{5}{\sqrt{100}} = 0.5. \text{ Since } n = 100 \text{ is large, the CLT applies.}$$

Approximately normal with mean 5 and standard deviation 0.5 minutes.

Exercise 12 - Is n large enough?

The CLT needs large n; for strong skew, n of at least 30 is a common guideline.

n = 8: no. n = 50: yes, approximately normal.

Exercise 13 - Difference of proportions

Mean: $0.40 - 0.30 = 0.10$.

$$\text{SD: } \sqrt{\frac{0.4(0.6)}{100} + \frac{0.3(0.7)}{120}} = \sqrt{0.0024 + 0.00175} = 0.0644.$$

Mean 0.10; standard deviation 0.0644.

Exercise 14 - Probability of a difference

$$z = \frac{0 - 0.10}{0.0644} = -1.55. \quad P(Z < -1.55) = 0.0606.$$

About 0.0606: a rare outcome.

Exercise 15 - Difference of means

$$\text{Mean: } 250 - 240 = 10. \quad \text{SD: } \sqrt{\frac{30^2}{40} + \frac{25^2}{50}} = \sqrt{22.5 + 12.5} = \sqrt{35} = 5.916.$$

Mean 10 ms; standard deviation 5.916 ms.

Exercise 16 - Sample size needed

$$\sqrt{\frac{0.5(0.5)}{n}} \leq 0.025, \text{ so } n \geq \frac{0.25}{0.000625} = 400.$$

n = 400.

Exercise 17 - Interpret the standard deviation

It measures typical sample-to-sample variation.

Sample proportions from random samples of 400 voters typically differ from the true proportion by about 0.025 (2.5 percentage points).

Exercise 18 - Within two standard deviations

$$\sigma_{\hat{p}} = \sqrt{\frac{0.5(0.5)}{100}} = 0.0490.$$

$$0.6 \pm 2(0.0490) = 0.6 \pm 0.098.$$

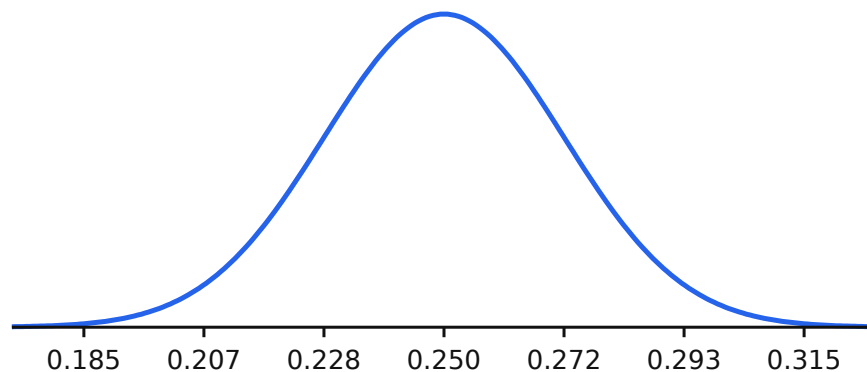
About 0.502 to 0.698.

Exercise 19 - Multi-step

$$\sigma_{\bar{x}} = \frac{0.4}{4} = 0.1. \quad z = \frac{16 - 16.1}{0.1} = -1. \quad P(Z < -1) = 0.1587.$$

0.1587.

Exercise 20 - Poll accuracy



Sampling distribution of $\hat{p}$, $n = 400$.

$$\sigma_{\hat{p}} = \sqrt{\frac{0.25(0.75)}{400}} = 0.0217. \quad z = \frac{0.22 - 0.25}{0.0217} = -1.39. \quad P(Z < -1.39) = 0.0823.$$

About 0.0823.