

AP Statistics - Significance Tests for a Proportion - Answer Key

Common Core AP STAT 6.4, AP STAT 6.5, AP STAT 6.6, AP STAT 6.7, HSS-IC.A.2, HSS-IC.B.5 · maths-pdf.fr/en/

Each solution follows State, Plan, Do, Conclude where relevant, with the hypotheses, conditions, test statistic, p-value, and a conclusion in context. P-values come from the normal table or normalcdf, rounded to four decimals.

AP Statistics · Common Core State Standards: **AP STAT 6.4, AP STAT 6.5, AP STAT 6.6, AP STAT 6.7, HSS-IC.A.2, HSS-IC.B.5**

Exercise 1 - Write hypotheses

(a) $H_0: p = 0.90$, $H_a: p < 0.90$, p is the true proportion of packages arriving on time. (b) $H_0: p = 0.5$, $H_a: p \neq 0.5$, p is the true proportion of heads. (c) $H_0: p = 0.60$, $H_a: p > 0.60$, p is the true proportion of students supporting block scheduling.

Hypotheses as shown.

Exercise 2 - Check conditions

Random: given. 10%: $120 < 250$. Large counts using $p_0 = 0.40$: $np_0 = 48$ and $n(1-p_0) = 72$, both at least 10.

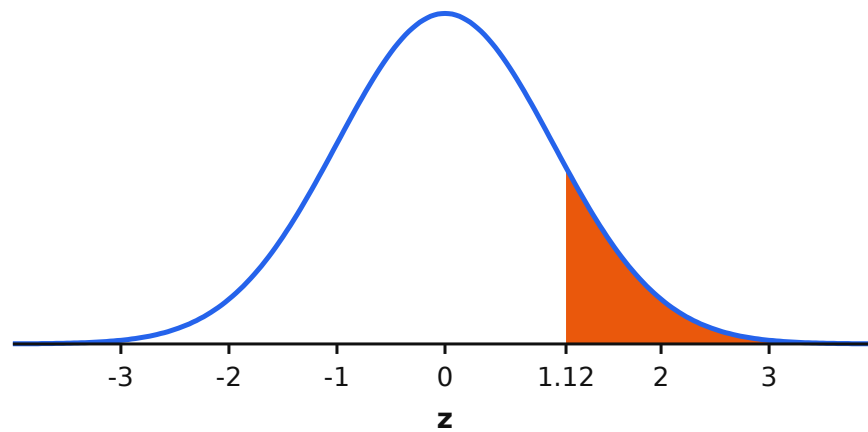
All conditions are satisfied.

Exercise 3 - Compute z

$$\hat{p} = \frac{54}{120} = 0.45. SE = \sqrt{\frac{0.4(0.6)}{120}} = 0.0447. z = \frac{0.45 - 0.40}{0.0447} = 1.12.$$

$z = 1.12$.

Exercise 4 - Find the p-value



P-value for $z = 1.12$.

$$P(Z > 1.12) = 1 - 0.8686 = 0.1314.$$

0.1314.

Exercise 5 - Conclude

$$0.1314 > 0.05.$$

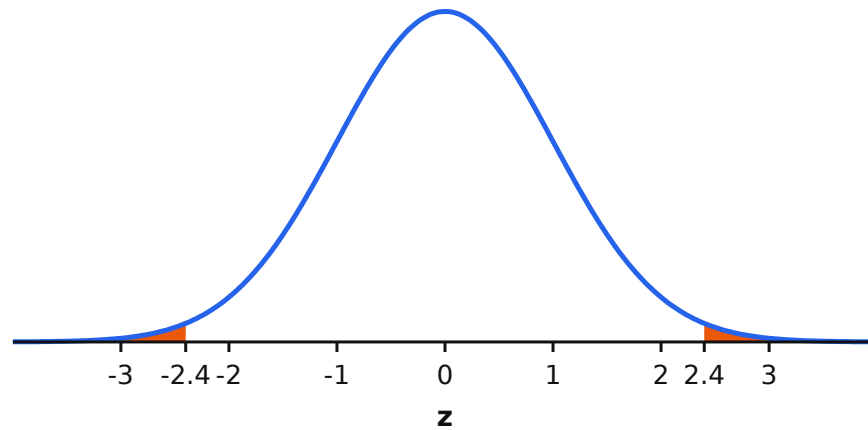
Because the p-value is greater than 0.05, we fail to reject H_0 . There is not convincing evidence that more than 40% of customers prefer self-checkout.

Exercise 6 - Interpret a p-value

The p-value is computed assuming H_0 is true.

If exactly 50% of voters favored the proposal, there would be a 2.1% chance of getting a sample proportion as far from 0.5 as the one observed, or farther, in a random sample of 400 voters.

Exercise 7 - Two-sided test



Two-sided p-value for $z = 2.4$.

$$z = \frac{0.62 - 0.5}{0.05} = 2.40. \text{ p-value} = 2P(Z > 2.40) = 0.0164.$$

0.0164 < 0.05: reject H_0 ; convincing evidence the coin is unfair.

Exercise 8 - Left-tailed test

$$\hat{p} = 0.92. SE = \sqrt{\frac{0.95(0.05)}{200}} = 0.0154. z = -1.95. \text{ p-value} = 0.0256.$$

Reject H_0 : convincing evidence that fewer than 95% of bulbs last over 1,000 hours.

Exercise 9 - Fail to reject

The p-value exceeds alpha.

We fail to reject H_0 . There is not convincing evidence that p differs from 0.5. We never accept H_0 , since the data could be consistent with other values.

Exercise 10 - Alpha and the conclusion

$$0.03 < 0.05 \text{ but } 0.03 > 0.01.$$

At alpha = 0.05, reject H_0 . At alpha = 0.01, fail to reject H_0 .

Exercise 11 - Confidence interval and test

0.50 lies outside the interval.

Reject H_0 : there is convincing evidence that p is not 0.50. (Intervals and two-sided tests give consistent conclusions.)

Exercise 12 - Type I error

	H_0 true	H_0 false
Reject H_0	Type I error	Correct (power)
Fail to reject H_0	Correct	Type II error

Type I and Type II errors.

A Type I error is rejecting a true H_0 .

Concluding the schedule increases attendance when it actually does not; the school might spend money on a schedule that has no effect.

Exercise 13 - Type II error

A Type II error fails to reject a false H_0 .

Concluding there is no convincing evidence that the new schedule increases attendance when it really does; the school misses a beneficial change.

Exercise 14 - Probability of Type I

The probability of a Type I error equals the significance level.

0.10.

Exercise 15 - Increase power

Larger n reduces the standard error, so the sampling distribution is narrower and a true difference is easier to detect.

Increase the sample size; increase α (or study a larger true effect).

Exercise 16 - Which error is worse?

Type I: tell a healthy patient they may be sick. Type II: tell a sick patient they are fine.

Type II is usually more serious because a sick patient would not receive treatment, though a Type I error causes worry and extra testing.

Exercise 17 - Complete test

State: $H_0: p = 0.50$, $H_a: p < 0.50$.

Plan: one-proportion z test; random, 10%, $np_0 = 150$ and $n(1-p_0) = 150$ at least 10.

Do: $\hat{p} = 0.44$, $SE = \sqrt{0.25/300} = 0.0289$, $z = -2.08$, $p = 0.0188$.

Conclude: p-value < 0.05, so reject H_0 . There is convincing evidence that fewer than half of the city's voters favor the tax.

Exercise 18 - Sample size effect

$n = 100$: $z = \frac{-0.06}{0.05} = -1.20$. $n = 1000$: $z = \frac{-0.06}{0.0158} = -3.79$.

The larger sample gives a much larger $|z|$, so the same difference is more convincing with more data.

Exercise 19 - Statistical vs practical

Statistical significance tells us the difference is unlikely to be chance, not that it is large.

Statistically significant but very small (0.4 percentage points), so it has little practical importance.

Exercise 20 - Multi-step

$H_0: p = 0.60$, $H_a: p > 0.60$. $\hat{p} = 0.63$, $SE = \sqrt{0.24/400} = 0.0245$, $z = 1.22$. p-value = 0.1112.

Fail to reject H_0 at alpha = 0.01: the evidence is not convincing at that stricter level.