

AP Statistics - Inference for the Difference of Two Means - Lesson

Common Core AP STAT 7.6, AP STAT 7.7, AP STAT 7.8, AP STAT 7.9, HSS-IC.B.5 · maths-pdf.fr/en/

To compare two means from independent groups, use a two-sample t procedure with standard error $\sqrt{s_1^2/n_1 + s_2^2/n_2}$. The interval is the difference in sample means plus or minus t^* times that standard error. Check independence, the 10 percent condition, and normality or large samples. For matched data, use a paired t procedure on the differences.

AP Statistics · Common Core State Standards: **AP STAT 7.6, AP STAT 7.7, AP STAT 7.8, AP STAT 7.9, HSS-IC.B.5**

Key facts

- Parameter: $\mu_1 - \mu_2$; statistic: $\bar{x}_1 - \bar{x}_2$.
- $SE = \sqrt{s_1^2/n_1 + s_2^2/n_2}$; df from the calculator, or $\min(n_1 - 1, n_2 - 1)$ conservatively.
- Interval: $(\bar{x}_1 - \bar{x}_2) \pm t^* SE$; test: $t = (\bar{x}_1 - \bar{x}_2)/SE$ with $H_0: \mu_1 - \mu_2 = 0$.
- Conditions: independent random samples or random assignment; 10%; each group normal or $n \geq 30$.
- Same subjects measured twice or matched pairs: use a one-sample t on the differences.

What do we compare with two means?

Two-sample procedures compare the means μ_1 and μ_2 of two populations, or the mean responses of two treatments. The statistic is $\bar{x}_1 - \bar{x}_2$. The groups must be **independent**: different people in each group, with no pairing.

Group	n	Mean	Std dev
Method A (new)	35	78.4	9.2
Method B (old)	40	72.1	11.5

Scores in two independent classes taught with different methods.

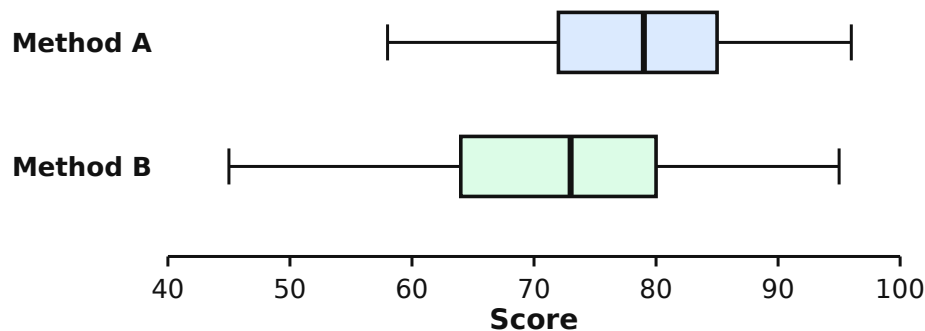
Example: two classes are taught with different methods. The difference in sample means is

$78.4 - 72.1 = 6.3$ points.

How do you build a two-sample t interval?

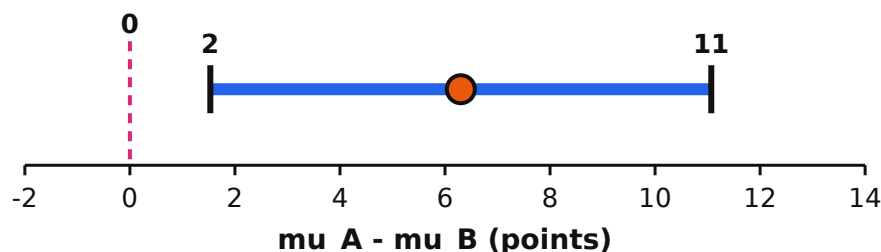
$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

The degrees of freedom come from the calculator (or the smaller of $n_1 - 1$ and $n_2 - 1$ as a safe conservative choice). Conditions: independent random samples or random assignment; 10% condition for each sample; and each group approximately normal or large (at least 30). Here both groups have at least 30 students.



Boxplots of both classes: no outliers and roughly symmetric.

$SE = \sqrt{\frac{9.2^2}{35} + \frac{11.5^2}{40}} = 2.393$, $df \approx 72.5$, $t^* = 1.993$, $ME = 4.769$. The 95% interval is (1.53, 11.07).



95% interval for $\mu_A - \mu_B$: (1.53, 11.07); 0 is not included.

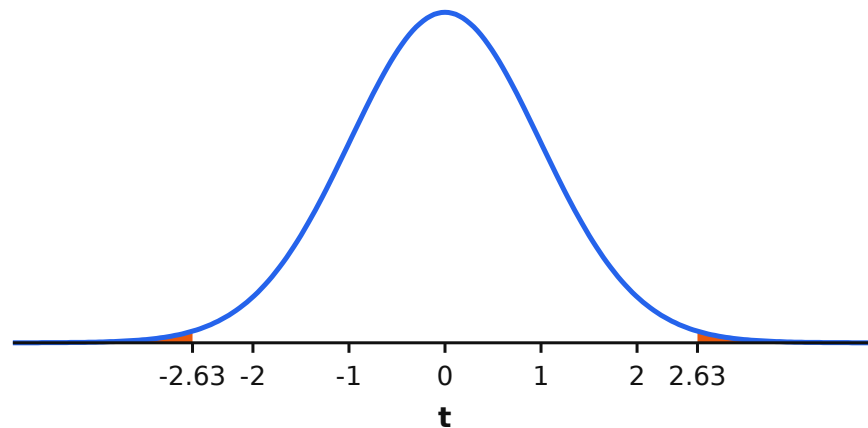
We are 95% confident that the interval from 1.53 to 11.07 points captures the true difference in mean scores between the populations taught with Method A and Method B. Since the whole interval is positive, Method A seems to give higher mean scores.

How do you test $H_0: \mu_1 = \mu_2$?

State $H_0: \mu_1 - \mu_2 = 0$ against $H_a: \mu_1 - \mu_2 \neq 0$ (or a one-sided alternative). The test statistic is:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

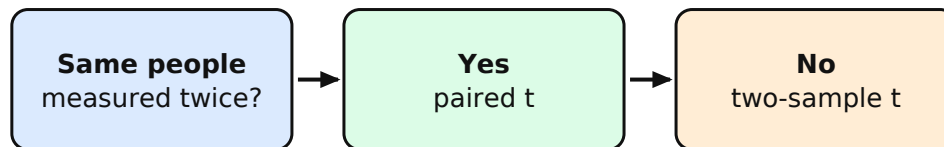
Here $t = \frac{6.3}{2.393} = 2.63$, and the two-sided p-value is 0.0104.



Two-sided p-value for $t = 2.63$: 0.0104.

Since 0.0104 is less than 0.05, we reject H_0 . There is convincing evidence of a difference in mean scores, and the interval tells us the size.

When is a paired procedure needed?



Choose the procedure by the design.

If the same students take both methods, or are matched in pairs, compute the differences and use a one-sample t procedure on them. Mixing up the two designs is the most common error.

Questions about inference for the difference of two means

How do I know whether to use a two-sample or a paired procedure?

Ask whether the two sets of measurements come from different individuals (two-sample) or from the same or matched individuals (paired). Pairing creates dependence, so the differences are analyzed as one sample.

How are the degrees of freedom found?

The calculator uses a formula that gives a non-integer df based on both standard deviations and sample sizes. If you work by hand, use the smaller of $n_1 - 1$ and $n_2 - 1$, which is conservative.

Why do the variances add under the square root?

The two samples are independent, so the variance of the difference is the sum of the variances. Standard deviations never add directly.

What does it mean when the interval for $\mu_1 - \mu_2$ contains 0?

It means 0 is a plausible value for the difference, so there is not convincing evidence that the two population means differ.

Do I need equal standard deviations?

No. The two-sample t procedures in AP Statistics do not assume equal variances; each group keeps its own standard deviation in the standard error.