

AP Statistics - Inference for the Difference of Two Proportions - Lesson

Common Core AP STAT 6.8, AP STAT 6.9, AP STAT 6.10, AP STAT 6.11, HSS-IC.B.5 · maths-pdf.fr/en/

To compare two proportions, estimate $p_1 - p_2$ with $\hat{p}_1 - \hat{p}_2$. A two-sample z interval adds the two sample variances under the square root. For $H_0: p_1 = p_2$, a z test uses the pooled proportion. Conditions: independent random samples or random assignment, the 10 percent condition, and at least 10 successes and failures in each group.

AP Statistics · Common Core State Standards: **AP STAT 6.8, AP STAT 6.9, AP STAT 6.10, AP STAT 6.11, HSS-IC.B.5**

Key facts

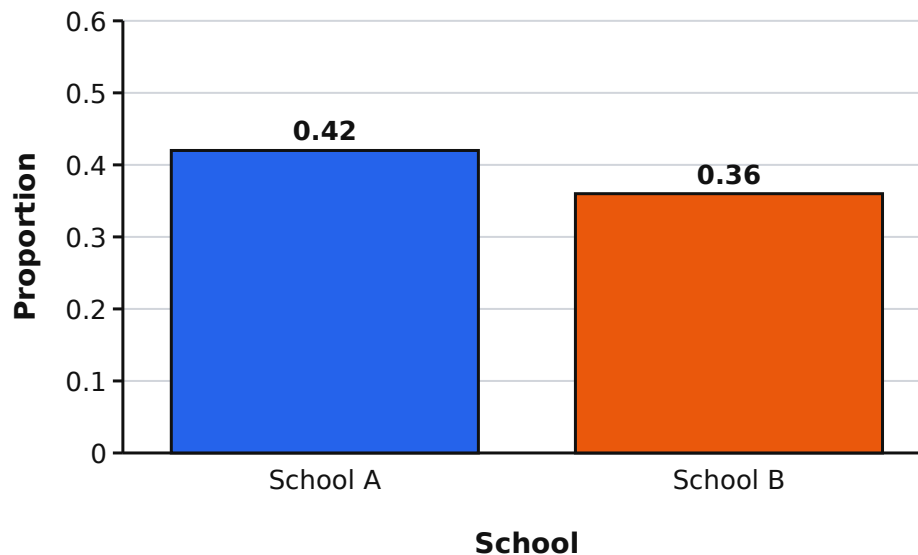
- Parameter: $p_1 - p_2$; statistic: $\hat{p}_1 - \hat{p}_2$.
- Interval: $(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\hat{p}_1(1 - \hat{p}_1)/n_1 + \hat{p}_2(1 - \hat{p}_2)/n_2}$.
- Test: pooled $\hat{p}_c = (x_1 + x_2)/(n_1 + n_2)$; $z = (\hat{p}_1 - \hat{p}_2)/\sqrt{\hat{p}_c(1 - \hat{p}_c)(1/n_1 + 1/n_2)}$.
- Conditions: independent random samples (or random assignment); 10% condition; at least 10 successes and 10 failures in each group.
- If the interval contains 0, a difference of 0 is plausible; if it does not, there is evidence of a difference.

What does inference about two proportions compare?

Often we compare the proportions of successes in two populations or in two treatment groups: p_1 and p_2 . The parameter of interest is the difference $p_1 - p_2$, estimated by $\hat{p}_1 - \hat{p}_2$. Example: random samples from two high schools are asked whether they support a later start time.

	Support	Do not support
School A	126	174
School B	90	160

Survey results for two independent random samples.



Sample proportions supporting the proposal.

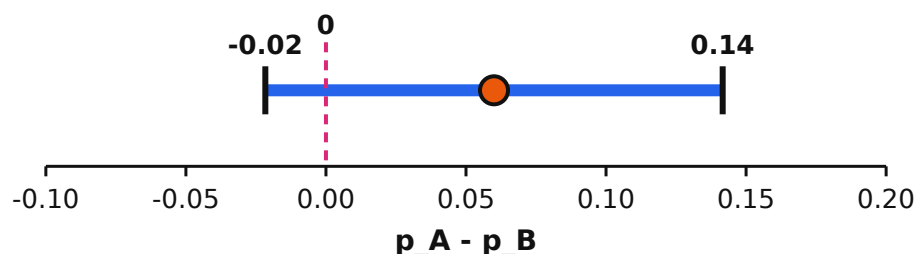
School A: $\hat{p}_1 = \frac{126}{300} = 0.42$. School B: $\hat{p}_2 = \frac{90}{250} = 0.36$. The difference is $\hat{p}_1 - \hat{p}_2 = 0.06$.

How do you build a confidence interval for $p_1 - p_2$?

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Conditions: two **independent** random samples (or random assignment to two groups), each sample under 10% of its population, and at least 10 successes and 10 failures *in each group*.

Here $SE = \sqrt{\frac{0.42(0.58)}{300} + \frac{0.36(0.64)}{250}} = 0.0416$, $ME = 1.96(0.0416) = 0.0816$, and the interval is $(-0.022, 0.142)$.



The 95% interval $(-0.022, 0.142)$ contains 0, so there is no convincing difference.

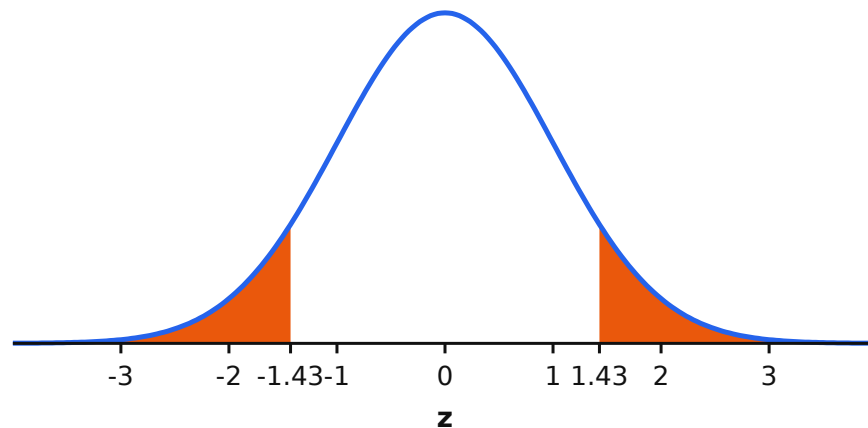
Interpretation: We are 95% confident that the interval from -0.022 to 0.142 captures the true difference (School A minus School B) in the proportions of students who support the proposal. Because the interval contains 0, a difference of 0 is plausible, so the data do not give convincing evidence of a difference.

How do you test $H_0: p_1 = p_2$?

When $H_0: p_1 = p_2$ is assumed, combine the groups into a **pooled proportion** $\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2}$ and use:

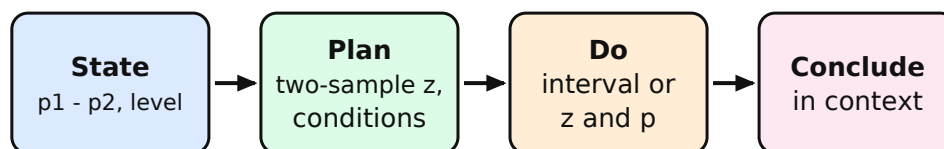
$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1 - \hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

For the surveys, $\hat{p}_c = \frac{216}{550} = 0.3927$, the standard error is 0.0418, $z = 1.43$, and the two-sided p-value is 0.1527.



Two-sided p-value for $z = 1.43$: 0.1527.

Since 0.1527 is greater than 0.05, we fail to reject H_0 . There is not convincing evidence that the proportions of supporters differ between the two schools. The test and the interval agree.



Four-step process.

The interval estimates the size of the difference; the test only measures how surprising it would be if there were none. Note that the interval uses each sample's own proportion, while the test pools them.

Questions about inference for the difference of two proportions

Why do we pool the proportions in a test but not in an interval?

A test assumes H_0 is true, which says the two proportions are equal, so combining the data

gives the best single estimate. An interval estimates the difference without assuming it is zero, so each sample keeps its own proportion.

What does it mean if the interval contains 0?

It means a difference of 0 is plausible, so there is not convincing evidence that the two population proportions differ. It does not prove that they are equal.

Can I use this procedure for a randomized experiment?

Yes. Instead of two random samples, you need random assignment of the units to the two treatments, and the counts of successes and failures in each group must still be at least 10.

How do I check the large counts condition for two groups?

Check that both x and $n - x$ are at least 10 for each group (using the sample counts for an interval and the pooled counts for a test).

How do I decide which proportion to put first?

Choose an order that makes the question easy to state, such as new minus old, and keep it throughout. Then interpret the sign of the interval in terms of that order.