

# Geometry - Arc Length and Sector Area - Lesson

Common Core G-C.B.5 · maths-pdf.fr/en/

**Arc length is the fraction  $\theta/360$  of the circumference, so  $s = (\theta/360)(2\pi r)$ . Sector area is the same fraction of the circle's area,  $A = (\theta/360)(\pi r^2)$ . One radian is the angle whose arc equals the radius, so 180 degrees equals  $\pi$  radians. In radians,  $s = r\theta$ .**

**Geometry** · Common Core State Standards: **G-C.B.5**

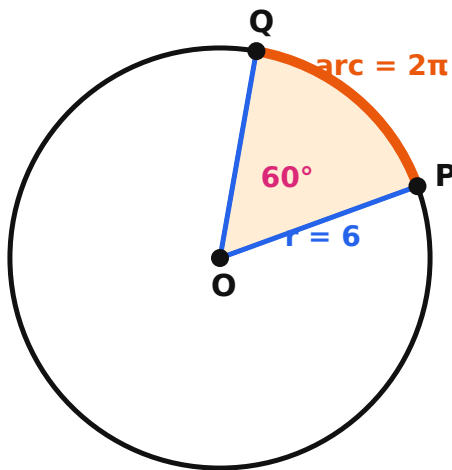
## Key facts

- Arc length =  $(\text{angle}/360) \times 2\pi r$ .
- Sector area =  $(\text{angle}/360) \times \pi r^2$ .
- 180 degrees =  $\pi$  radians.
- In radians: arc length  $s = r$  times  $\theta$ , area  $A = (1/2) r^2 \theta$ .
- A radian is the central angle whose arc length equals the radius.

## How do you find arc length?

An arc is a fraction of the whole circle. A central angle of  $\theta$  degrees is  $\frac{\theta}{360}$  of a full turn, so its arc length is that fraction of the circumference.

$$s = \frac{\theta}{360} \cdot 2\pi r$$

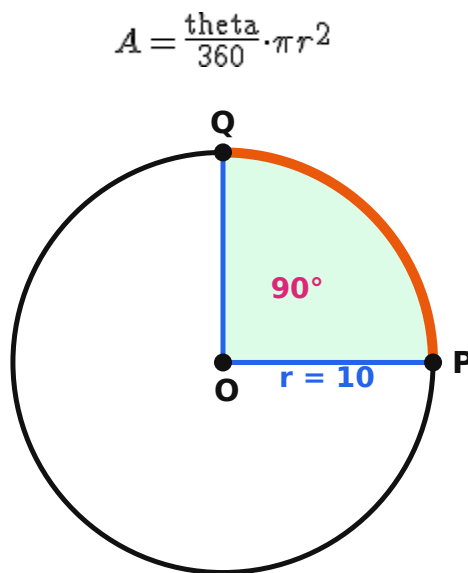


$$\text{Arc length} = (60/360) \text{ of the circumference} = (1/6)(2\pi \cdot 6) = 2\pi.$$

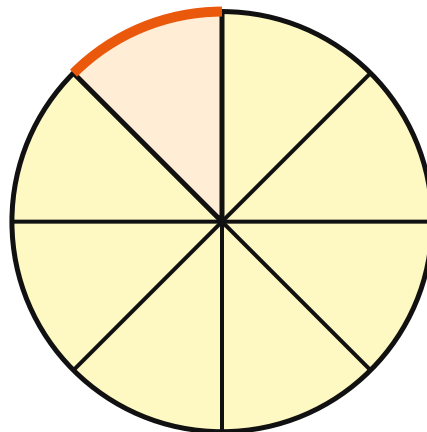
For  $r=6$  and  $\theta=60^\circ$ :  $s=\frac{60}{360}\cdot 2\pi\cdot 6=2\pi=6.28$  units, which is about 6.28.

## How do you find the area of a sector?

A **sector** is the region between two radii and an arc. Its area is the same fraction of the circle's area.



Sector area =  $(90/360)$  of the circle's area =  $25\pi$ .

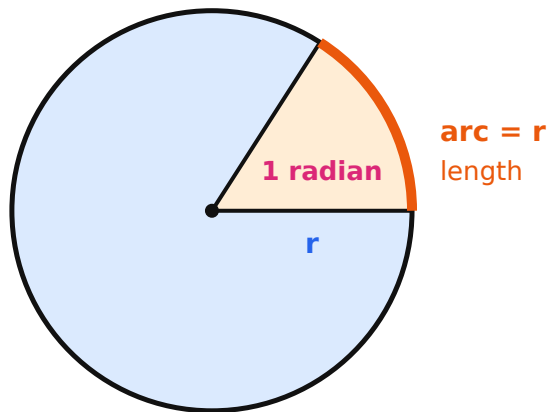


A pizza cut into 8 equal slices: each slice is a sector of  $45^\circ$ .

## What is a radian?

One **radian** is the measure of a central angle whose arc has the same length as the radius. A full circle is  $2\pi$  radians, so  $180^\circ = \pi$  radians. In radian measure, arc length and sector area are

simple:  $s = r\theta$  and  $A = \frac{1}{2}r^2\theta$ .



One radian: the arc has the same length as the radius.

| Degrees     | Radians         |
|-------------|-----------------|
| $30^\circ$  | $\frac{\pi}{6}$ |
| $45^\circ$  | $\frac{\pi}{4}$ |
| $60^\circ$  | $\frac{\pi}{3}$ |
| $90^\circ$  | $\frac{\pi}{2}$ |
| $180^\circ$ | $\pi$           |

To convert, multiply degrees by  $\frac{\pi}{180}$  to get radians, or multiply radians by  $\frac{180}{\pi}$  to get degrees.

### How do you solve an arc or sector problem?

1. Write the fraction of the circle: angle divided by 360 (or by  $2\pi$ ).
2. Multiply by the circumference for arc length or by the area for a sector.
3. Keep pi in the answer or round at the end, and include units (square units for area).

**Example:** A sector with radius 8 and central angle  $45^\circ$  has area  $\frac{45}{360} \cdot \pi \cdot 8^2 = 8\pi$ , about 25.13 square units.

### Questions about arc length and sector area

#### How do you find the length of an arc?

Multiply the circumference by the fraction of the circle: angle divided by 360. For radius 6 and 60 degrees the arc length is  $2\pi$ , about 6.28.

## What is a sector?

A sector is the region bounded by two radii and the arc between them, like a slice of pizza. Its area is the same fraction of the circle's area as its angle is of 360 degrees.

## What is a radian?

A radian is the angle at the center of a circle that intercepts an arc equal in length to the radius. About 57.3 degrees make one radian.

## How do you convert degrees to radians?

Multiply by pi over 180. For example, 135 degrees is  $\frac{3\pi}{4}$  radians.

## Why is the area of a sector one half $r^2\theta$ ?

In radians the fraction of the circle is  $\frac{\theta}{2\pi}$ , so the area is  $(\frac{\theta}{2\pi})(\pi r^2) = \frac{1}{2} r^2 \theta$ .