

Geometry - Area, Volume and Dissection Arguments - Lesson

Common Core G-GMD.A.1, G-GMD.A.3 · maths-pdf.fr/en/

A circle has area πr^2 . A prism or cylinder has volume Bh , a pyramid or cone has one third of that, and a sphere has volume four thirds πr^3 . Cavalieri's principle shows that solids with equal heights and equal cross-sections at every level have equal volumes.

Geometry · Common Core State Standards: **G-GMD.A.1, G-GMD.A.3**

Key facts

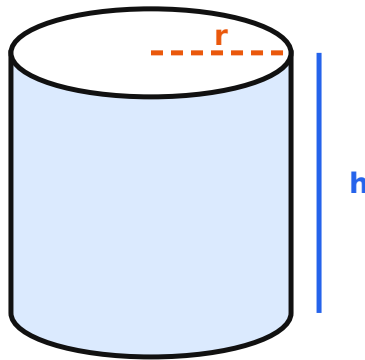
- Circle: $C = 2\pi r$, $A = \pi r^2$.
- Prism and cylinder: $V = Bh$.
- Pyramid and cone: $V = (1/3)Bh$.
- Sphere: $V = (4/3)\pi r^3$.
- Cavalieri's principle: same height and same cross-sectional areas give the same volume.

What are the circle formulas?

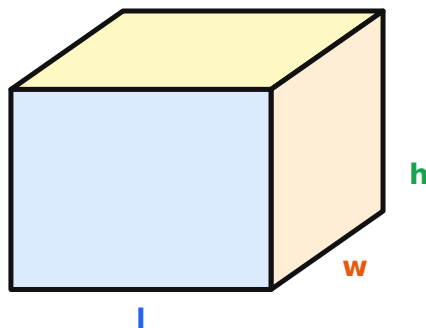
The circumference of a circle is $C = 2\pi r$ and its area is $A = \pi r^2$. You can see the area formula by cutting a circle into thin sectors and rearranging them into a shape close to a rectangle with width πr and height r . As the sectors get thinner, the rearranged shape gets closer to a rectangle, a dissection argument for $A = \pi r \cdot r$.

How do you find the volume of a prism or a cylinder?

The volume of any prism or cylinder is the area of the base times the height: $V = Bh$. For a cylinder the base is a circle, so $V = \pi r^2 h$.



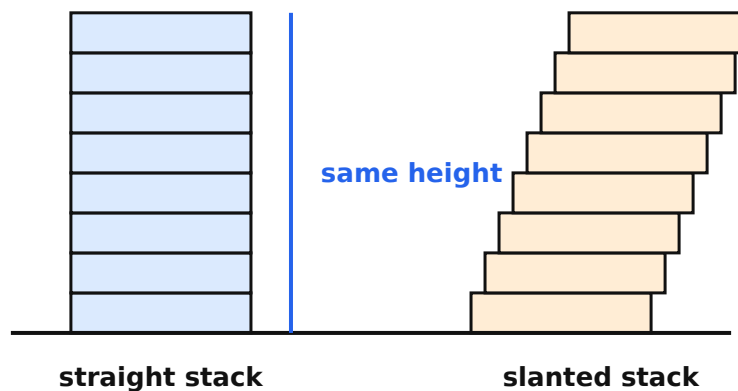
Cylinder: $V = \pi r^2 h$.



Prism: $V = Bh$.

What is Cavalieri's principle?

Cavalieri's principle says that two solids with the same height and the same cross-sectional area at every level have the same volume. A slanted stack of coins has the same volume as a straight stack of the same coins.

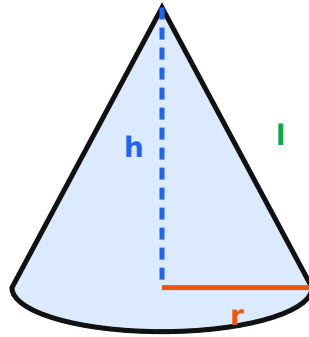


Cavalieri's principle: equal heights and equal cross-sections give equal volumes.

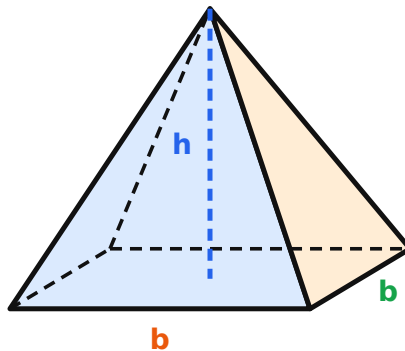
How do you find the volume of a pyramid or a cone?

A pyramid or cone has one third of the volume of the prism or cylinder with the same base and height. For a cube, three congruent pyramids fit together, each with a face for its base and the opposite corner as its apex.

$$V_{\text{pyramid}} = \frac{1}{3}Bh \quad V_{\text{cone}} = \frac{1}{3}\pi r^2h$$



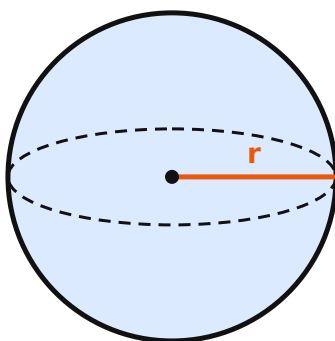
Cone: $V = \frac{1}{3}\pi r^2h$.



Pyramid: $V = \frac{1}{3}Bh$.

What is the volume of a sphere?

A sphere of radius r has $V = \frac{4}{3}\pi r^3$. Cavalieri's principle compares a hemisphere to a cylinder with a cone removed to prove this.



Sphere: $V = \frac{4}{3}\pi r^3$.

1. Identify the solid and its dimensions (use radius, not diameter).
2. Choose the formula: Bh for prisms and cylinders, one third Bh for pyramids and cones, four thirds πr^3 for spheres.
3. Substitute and calculate. Keep π until the end.
4. Write cubic units.

Example: A cone has radius 6 and height 9. $V = \frac{1}{3}\pi \cdot 6^2 \cdot 9 = 108\pi$, about 339.3 cubic units.

Questions about area, volume and dissection arguments

What is the volume formula for a cylinder?

$V = \pi r^2 h$, the area of the circular base times the height. A cylinder with radius 3 and height 10 has volume 90π , about 282.7 cubic units.

Why does a cone have one third of the volume of a cylinder?

Three congruent pyramids fill a cube, and the same relationship holds for cones and cylinders with equal base and height.

What is Cavalieri's principle?

If two solids have the same height and the same cross-sectional area at every level, they have the same volume, even when one is slanted.

How do you find the volume of a sphere?

Use $V = \frac{4}{3}\pi r^3$. A sphere of radius 3 has volume 36π , about 113.1 cubic units.

Which measure do you use in the formulas, radius or diameter?

Use the radius. If you are given a diameter, divide it by 2 first.