

Geometry - Circles: Angles, Chords and Tangents - Lesson

Common Core G-C.A.1, G-C.A.2, G-C.A.3, G-C.A.4 · maths-pdf.fr/en/

A central angle equals its intercepted arc, and an inscribed angle is half of its intercepted arc. An angle inscribed in a semicircle is a right angle, and opposite angles of an inscribed quadrilateral add to 180 degrees. A tangent is perpendicular to the radius at the point of tangency, and two tangent segments from one point are equal.

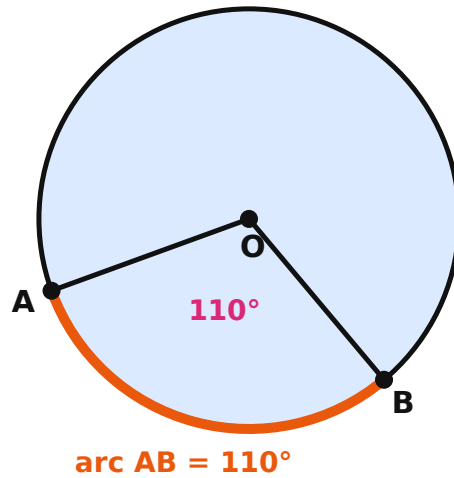
Geometry · Common Core State Standards: **G-C.A.1, G-C.A.2, G-C.A.3, G-C.A.4**

Key facts

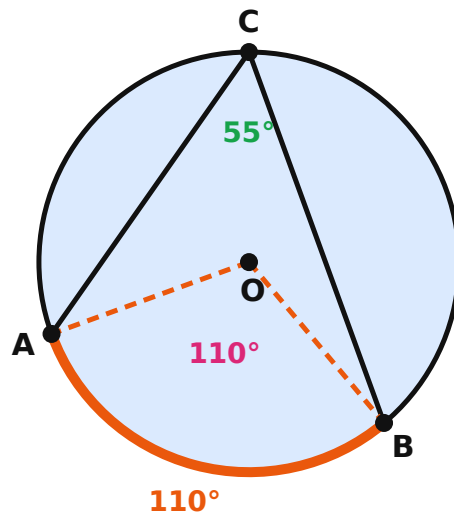
- Central angle = intercepted arc; inscribed angle = half the arc.
- An angle inscribed in a semicircle measures 90 degrees.
- Opposite angles of an inscribed quadrilateral are supplementary.
- A tangent is perpendicular to the radius at the point of tangency.
- Tangent segments from one outside point are congruent.

What are central and inscribed angles?

A **central angle** has its vertex at the center of the circle. Its measure equals the measure of its intercepted arc. An **inscribed angle** has its vertex on the circle, and its measure is half of its intercepted arc.

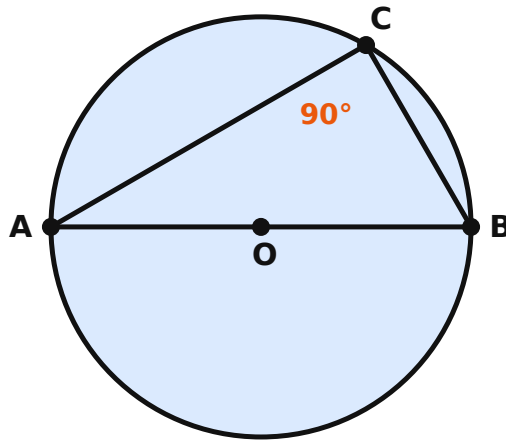


A central angle equals its intercepted arc.



An inscribed angle is half of its intercepted arc: $110/2 = 55$.

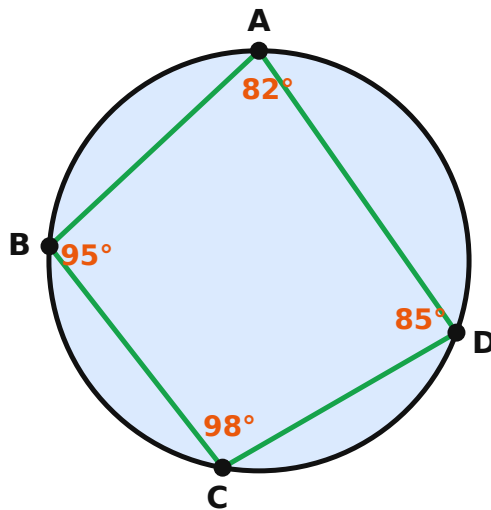
- Inscribed angles that intercept the same arc are congruent.
- An angle inscribed in a semicircle is a right angle.



An angle inscribed in a semicircle is a right angle.

What is true about an inscribed quadrilateral?

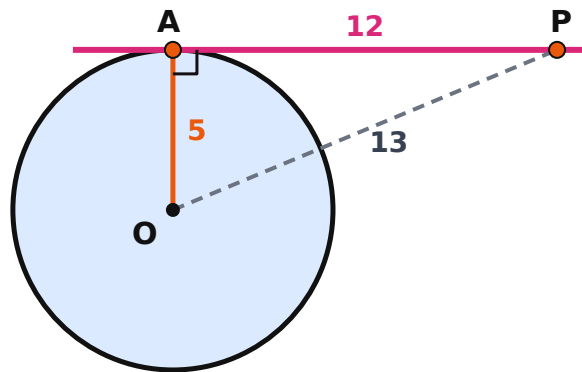
When all four vertices of a quadrilateral lie on a circle, opposite angles are supplementary. Each angle is half of the arc opposite it, and the two opposite arcs together make 360° .



Opposite angles of an inscribed quadrilateral are supplementary.

What are the properties of tangents?

A **tangent** touches the circle at exactly one point. It is perpendicular to the radius drawn to that point. Two tangent segments from the same outside point have equal length.



A tangent is perpendicular to the radius at the point of tangency.

Tangent facts

In the figure, $\angle OAP = 90^\circ$, so $OP^2 = OA^2 + PA^2$. Here $5^2 + 12^2 = 13^2$.

What about chords, circumscribed circles and similarity?

A radius perpendicular to a chord bisects the chord. The center of the circle that passes through all three vertices of a triangle (the circumcenter) is where the perpendicular bisectors meet. The center of the inscribed circle (the incenter) is where the angle bisectors meet. All circles are similar: a dilation maps any circle to any other, so the ratio of circumference to diameter is always π .

How do you solve a circle angle problem?

1. Mark central angles, inscribed angles and arcs on a sketch.
2. Pick the relationship: central = arc, inscribed = half the arc.
3. Write an equation and solve.
4. Check that the arcs of a circle add to 360° .

Example: An inscribed angle is $(x+20)^\circ$ and the central angle on the same arc is $(4x-30)^\circ$. Then $4x-30 = 2(x+20)$, so $x = 35$ and the inscribed angle is 55° .

Questions about circles: angles, chords and tangents

What is the difference between a central angle and an inscribed angle?

A central angle has its vertex at the center and equals its arc. An inscribed angle has its vertex on the circle and is half of its intercepted arc.

Why is an angle in a semicircle a right angle?

The intercepted arc is a semicircle of 180 degrees, so the inscribed angle is half of that, 90 degrees.

Why are opposite angles of an inscribed quadrilateral supplementary?

Each angle is half of the arc opposite it, and the two opposite arcs add to the full circle, 360 degrees, so the angles add to 180 degrees.

How do you know a line is tangent to a circle?

A line is tangent if it is perpendicular to a radius at the point where it meets the circle. For example, a segment of 12 from a point 13 units from the center of a circle of radius 5 is tangent.

Why are all circles similar?

A translation moves one center onto the other, and a dilation with the ratio of the radii maps one circle onto the other.