

Geometry - Equations of Circles and Parabolas - Lesson

Common Core G-GPE.A.1, G-GPE.A.2 · maths-pdf.fr/en/

The graph of $(x - h)^2 + (y - k)^2 = r^2$ is a circle with center (h, k) and radius r , because the equation says every point is r units from the center. Expanded equations are converted by completing the square. A parabola is the set of points equally far from a focus and a directrix.

Geometry · Common Core State Standards: **G-GPE.A.1, G-GPE.A.2**

Key facts

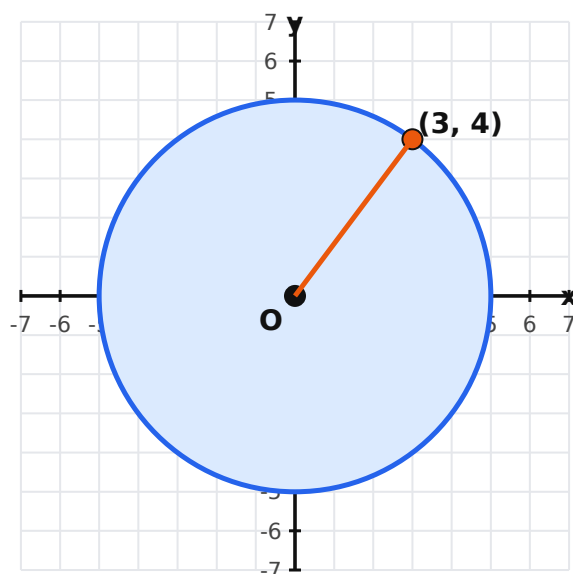
- Circle: $(x - h)^2 + (y - k)^2 = r^2$, center (h, k) , radius r .
- Read the center with opposite signs from the equation.
- Inside, on, or outside: compare $(x - h)^2 + (y - k)^2$ with r^2 .
- Complete the square to convert $x^2 + y^2 + Dx + Ey + F = 0$.
- Parabola with focus $(0, p)$ and directrix $y = -p$: $x^2 = 4py$.

How is a circle written as an equation?

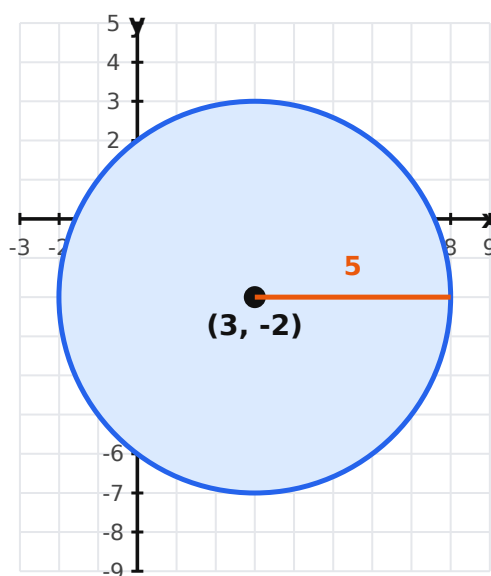
Choose a center (h, k) and a radius r . A point (x, y) sits on the circle exactly when its distance to the center is r . Squaring the distance formula removes the root and leaves the equation below.

$$(x - h)^2 + (y - k)^2 = r^2$$

When the center is the origin, the equation shrinks to $x^2 + y^2 = r^2$. Take $x^2 + y^2 = 25$: the point $(3, 4)$ fits, since $9 + 16 = 25$, so it lies on that circle.



Origin-centered circle of radius 5 with the point (3, 4) on it.



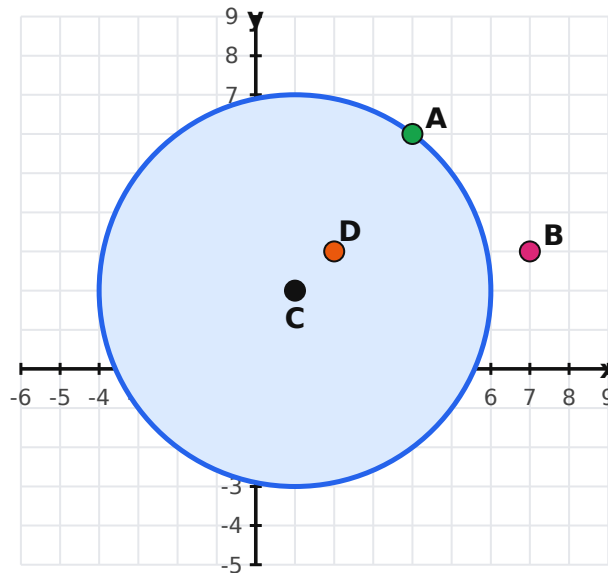
Circle drawn around (3, -2) with radius 5.

Where are the center and the radius in the equation?

The signs inside the parentheses are opposite to the center's coordinates. Reading $(x+4)^2+(y-1)^2=36$ gives the center $(-4,1)$, and taking the square root of 36 gives the radius 6.

Is a given point on the circle, inside it or outside it?

Put the coordinates of the point into the left side of the equation and compare the result with r^2 : equal means the point is on the circle, smaller means inside, and larger means outside.



Point A lies on the circle, B lies outside and D lies inside.

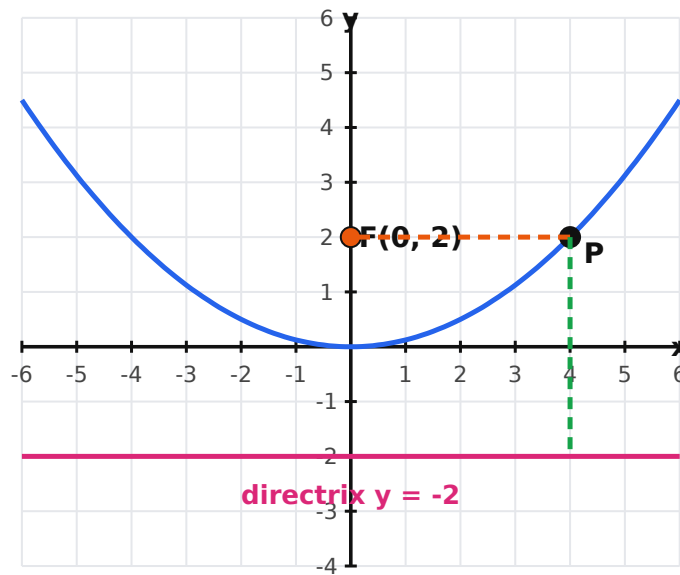
What if the equation is not in center-radius form?

Sometimes a circle arrives expanded, as $x^2 + y^2 + Dx + Ey + F = 0$. Collect the x terms together and the y terms together, add the needed squares to both sides, and factor each group to get back to the center-radius form.

1. Send the constant term to the right side and collect the x terms and the y terms.
2. Add $\left(\frac{D}{2}\right)^2$ and $\left(\frac{E}{2}\right)^2$ to both sides.
3. Write each group as a perfect square.
4. State the center and the radius from the result.

How do a focus and a directrix define a parabola?

Take a point called the **focus** and a line called the **directrix**. The **parabola** they define consists of every point whose distances to the two are equal. Place the focus at $(0, p)$ and the directrix along $y = -p$. Setting the two distances equal and simplifying gives $x^2 = 4py$.



Each point of the curve is the same distance from F and from the line $y = -2$.

Worked example: To convert $x^2 + y^2 - 6x + 4y - 12 = 0$: $(x-3)^2 + (y+2)^2 = 25$. From this form the center is $(3, -2)$ and the radius is 5.

Questions about equations of circles and parabolas

How do you write the equation of a circle?

Use $(x - h)^2 + (y - k)^2 = r^2$ with the center (h, k) and radius r . A circle with center $(3, -2)$ and radius 5 has equation $(x - 3)^2 + (y + 2)^2 = 25$.

How do you find the center and radius from an equation?

Read h and k with opposite signs and take the square root of the right side. From $(x + 4)^2 + (y - 1)^2 = 36$ the center is $(-4, 1)$ and the radius is 6.

How do you complete the square for a circle?

Group x terms and y terms, add half the coefficient squared to each group and to the other side, then factor. Both x and y need the step.

What is a parabola?

A parabola is the set of points that are the same distance from a focus and from a directrix line. For focus $(0, 2)$ and directrix $y = -2$ the equation is $x^2 = 8y$.

How do you tell if a point is inside a circle?

Substitute it into $(x - h)^2 + (y - k)^2$. If the result is less than r^2 , the point is inside the circle.