

Geometry - Equations of Circles and Parabolas - Answer Key

Common Core G-GPE.A.1, G-GPE.A.2 · maths-pdf.fr/en/

Use this answer key to check the 20 problems on equations of circles. Every solution shows the formula or theorem used, each step of the work, and the final answer in bold. If an answer differs from yours, compare the steps line by line to find where the work changed.

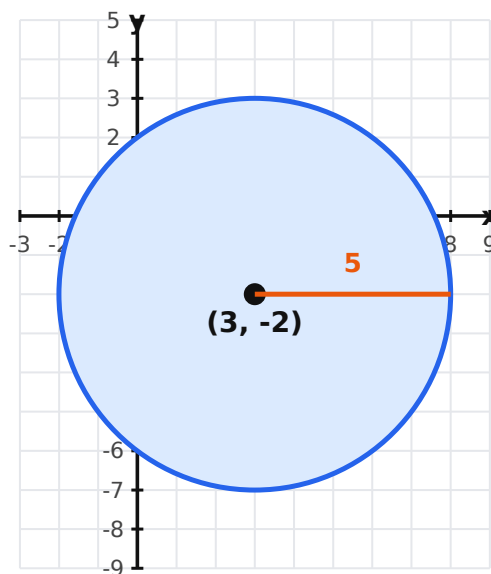
Geometry · Common Core State Standards: **G-GPE.A.1, G-GPE.A.2**

Exercise 1 - Circle at the origin

$$x^2 + y^2 = 7^2$$

$$x^2 + y^2 = 49$$

Exercise 2 - Center and radius given



Circle drawn around (3, -2) with radius 5.

$$(x-3)^2 + (y-(-2))^2 = 5^2$$

$$(x-3)^2 + (y+2)^2 = 25$$

Exercise 3 - Read the equation

The center has opposite signs: $(-4, 1)$. The radius is $\sqrt{36} = 6$.

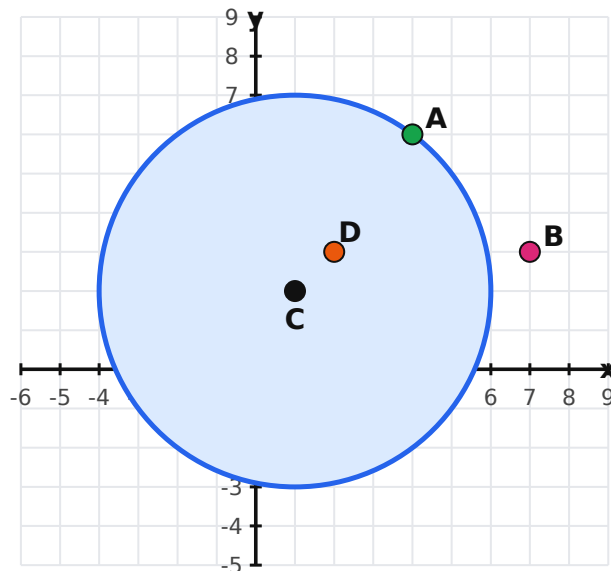
Center $(-4, 1)$; radius 6

Exercise 4 - Radius from r squared

$$r = \sqrt{20} = 2\sqrt{5}$$

$$2\sqrt{5}$$

Exercise 5 - Point on the circle?



Point A lies on the circle, B lies outside and D lies inside.

$$(4-1)^2 + (6-2)^2 = 9 + 16 = 25.$$

Yes

Exercise 6 - Outside or inside?

$$B: 6^2 + 1^2 = 37 > 25, \text{ outside.}$$

$$D: 1^2 + 1^2 = 2 < 25, \text{ inside.}$$

B is outside; D is inside

Exercise 7 - Circle through a point

$$r^2 = (3+1)^2 + (5-2)^2 = 16+9 = 25.$$

$$(x + 1)^2 + (y - 2)^2 = 25$$

Exercise 8 - Circle from a diameter

Center = midpoint: (2,5).

Diameter: $d^2 = 8^2 + 4^2 = 80$, so $r^2 = \frac{80}{4} = 20$.

$$(x - 2)^2 + (y - 5)^2 = 20$$

Exercise 9 - Complete the square (1)

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$$

$$(x-3)^2 + (y+2)^2 = 25$$

Center (3, -2); radius 5

Exercise 10 - Complete the square (2)

$$(x^2 - 12x + 36) + (y^2 + 6y + 9) = -20 + 36 + 9$$

$$(x-6)^2 + (y+3)^2 = 25$$

Center (6, -3); radius 5

Exercise 11 - Complete the square (3)

$$(x^2 - 10x + 25) + y^2 = 25$$

$$(x-5)^2 + y^2 = 25$$

Center (5, 0); radius 5

Exercise 12 - Tangent to the x-axis

The radius equals the distance to the x-axis: $r = 3$.

$$(x-4)^2 + (y-3)^2 = 9$$

The point of tangency is directly below the center: $(4,0)$.

$$(x - 4)^2 + (y - 3)^2 = 9; (4, 0)$$

Exercise 13 - Line and circle

Substitute: $x^2 + (x+1)^2 = 25$.

$$2x^2 + 2x - 24 = 0, \text{ so } x^2 + x - 12 = 0 \text{ and } (x+4)(x-3) = 0.$$

$$x = 3 \text{ gives } (3, 4); x = -4 \text{ gives } (-4, -3).$$

$$(3, 4) \text{ and } (-4, -3)$$

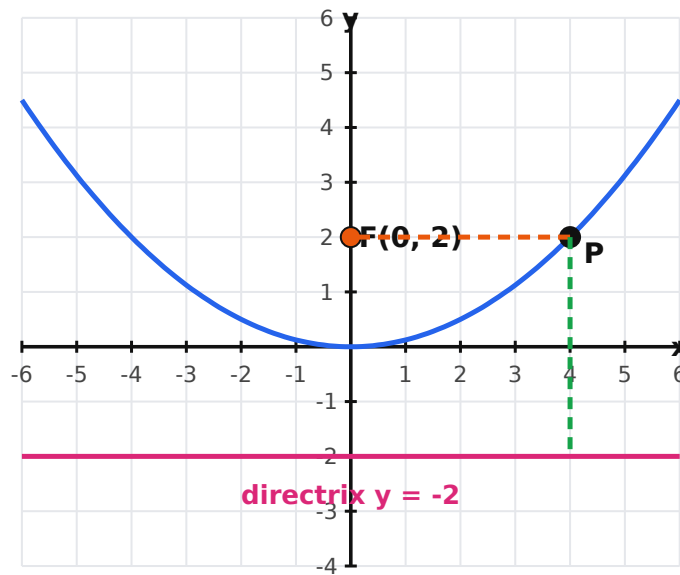
Exercise 14 - Area and circumference

$$r = 9.$$

$$C = 2\pi \cdot 9 = 18\pi \text{ and } A = \pi \cdot 9^2 = 81\pi.$$

$$18\pi \text{ and } 81\pi$$

Exercise 15 - Parabola equation



Each point of the curve is the same distance from F and from the line $y = -2$.

A point (x, y) is equally far from the focus and the directrix.

$$\sqrt{x^2 + (y-2)^2} = y+2$$

$$x^2 + y^2 - 4y + 4 = y^2 + 4y + 4$$

$$x^2 = 8y$$

$$\mathbf{x^2 = 8y}$$

Exercise 16 - Parabola with focus (0, 3)

Use $x^2 = 4py$ with $p = 3$.

$$\mathbf{x^2 = 12y}$$

Exercise 17 - Check a point on a parabola

$4^2 = 16 = 8 \cdot 2$, so P is on the parabola.

Distance to the focus: 4. Distance to the directrix: $2 - (-2) = 4$. They are equal.

Both distances are 4

Exercise 18 - Find the focus

Write $x^2 = 16y$, so $4p = 16$ and $p = 4$.

Focus (0, 4); directrix $y = -4$

Exercise 19 - Radio coverage

$$(x-10)^2 + (y+5)^2 = 169.$$

$$\text{City: } (22-10)^2 + (0+5)^2 = 144 + 25 = 169.$$

$(x - 10)^2 + (y + 5)^2 = 169$; the city is exactly on the boundary

Exercise 20 - Satellite dish

$x^2 = 24y$, so $4p = 24$ and $p = 6$: the focus is (0,6), 6 feet above the vertex.

At width 24, $x = 12$ and $y = \frac{144}{24} = 6$.

Receiver at (0, 6); depth 6 ft