

Geometry - Congruent Triangles and Rigid Motions - Lesson

Common Core G-CO.B.7, G-CO.B.8 · maths-pdf.fr/en/

Two triangles are congruent when a sequence of rigid motions maps one onto the other, so all corresponding sides and angles are equal. You can prove triangles congruent with SSS, SAS, ASA, AAS or HL. Once triangles are congruent, CPCTC says their corresponding parts are congruent. SSA and AAA are not valid tests.

Geometry · Common Core State Standards: **G-CO.B.7, G-CO.B.8**

Key facts

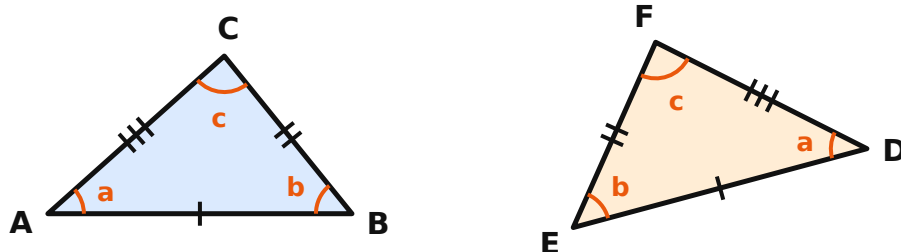
- Congruent triangles have three pairs of congruent sides and three pairs of congruent angles.
- Valid tests: SSS, SAS, ASA, AAS and, for right triangles, HL.
- SSA and AAA do not prove congruence.
- CPCTC: after proving triangles congruent, corresponding parts are congruent.
- A rigid motion maps a figure onto a congruent figure.

What does congruent mean?

Two figures are **congruent** when a sequence of rigid motions maps one onto the other. Rigid motions are translations, reflections and rotations, and they preserve distance and angle measure. So congruent figures have the same size and the same shape.

Corresponding parts

If triangle ABC is congruent to triangle DEF, then the vertices match in order: A with D, B with E, C with F. Corresponding sides and corresponding angles are congruent. We often cite this as **CPCTC**: corresponding parts of congruent triangles are congruent.

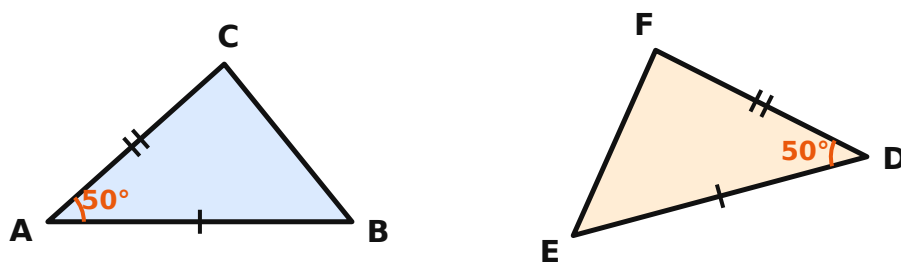


Matching tick marks and matching angle letters show corresponding parts.

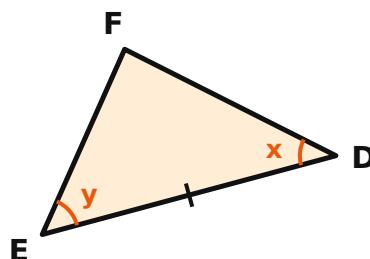
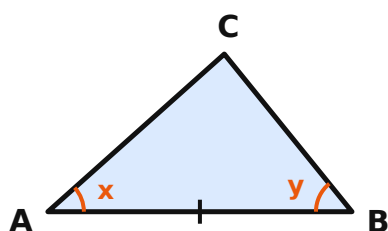
Which tests prove that two triangles are congruent?

You do not need to check all six pairs of parts. Five shortcuts are enough.

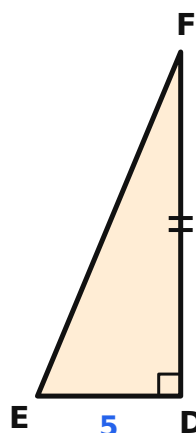
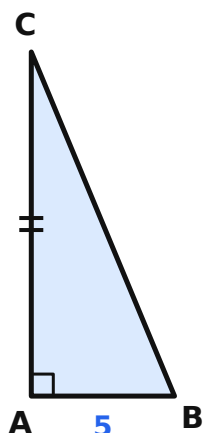
Test	What you need
SSS	three pairs of congruent sides
SAS	two pairs of sides and the angle between them
ASA	two pairs of angles and the side between them
AAS	two pairs of angles and a side that is not between them
HL	right triangles with congruent hypotenuses and one pair of congruent legs



SAS: two sides and the angle between them.



ASA: two angles and the side between them.



HL: right triangles with congruent hypotenuse and a congruent leg.

The pattern **SSA** does not work, because two different triangles can share two sides and a non-included angle. AAA does not work either, because it only guarantees the same shape, not the same size.

How do you decide whether two triangles are congruent?

1. Mark the given information on the diagram, including shared sides and vertical angles.
2. List the three pairs of parts, matching vertices in order.
3. Look for SSS, SAS, ASA, AAS or HL.
4. If one test fits, write the congruence statement and use CPCTC to find the missing part.

Example: Triangles ABC and DEF have $AB = DE = 7$, $AC = DF = 9$ and $m\angle A = m\angle D = 50^\circ$. The angle is between the two sides, so the triangles are congruent by SAS. By CPCTC, $BC = EF$.

Questions about congruent triangles and rigid motions

What is the difference between congruent and similar?

Congruent figures have the same size and shape, so a rigid motion maps one onto the other. Similar figures have the same shape but may have different sizes.

What does CPCTC stand for?

It stands for corresponding parts of congruent triangles are congruent. You use it after proving two triangles congruent to show that a pair of sides or angles is equal.

Why does SSA not prove congruence?

With two sides and a non-included angle, the third side can sometimes swing to two positions. That gives two different triangles with the same given parts.

When can you use HL?

Use HL only for right triangles. You need congruent hypotenuses and one pair of congruent legs.

How do rigid motions show that triangles are congruent?

If a translation, reflection, rotation or combination maps every vertex of one triangle onto the corresponding vertex of the other, distances and angles are preserved, so the triangles are congruent.