

Geometry - Geometric Constructions - Lesson

Common Core G-CO.D.12, G-CO.D.13 · maths-pdf.fr/en/

A geometric construction uses only a compass and a straightedge. You can bisect a segment or an angle, build perpendicular and parallel lines, and construct an equilateral triangle, a square or a regular hexagon inscribed in a circle. Each step is justified by congruent triangles or by equal radii, so every construction is also a short proof.

Geometry · Common Core State Standards: **G-CO.D.12, G-CO.D.13**

Key facts

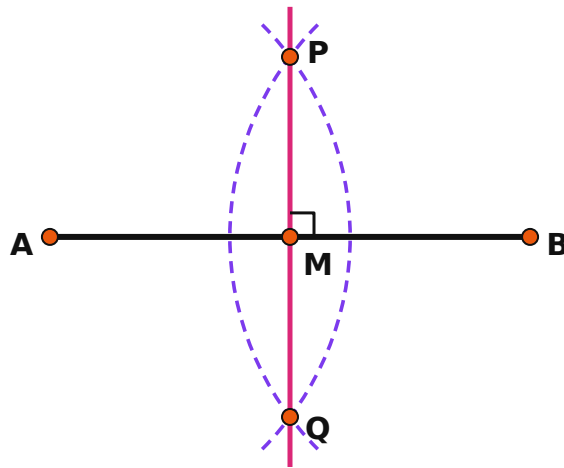
- Constructions use only a compass and a straightedge.
- Perpendicular bisector: equal arcs from both endpoints with radius more than half the segment.
- Angle bisector: arc at the vertex, then equal arcs from the two crossing points.
- Equilateral triangle: arcs of radius AB from A and from B.
- A regular hexagon has side length equal to the radius of its circumscribed circle.

What tools are used in geometric constructions?

A **construction** uses only a compass and a straightedge, with no measuring. The compass makes circles and arcs of a fixed radius, and the straightedge draws lines through two points. Each step works because it creates congruent segments or triangles.

How do you construct a perpendicular bisector?

1. Place the compass on A and open it to more than half of AB. Draw an arc above and below AB.
2. Keep the same radius, place the compass on B, and draw arcs that cross the first two.
3. Join the two crossing points P and Q. The line PQ is the perpendicular bisector, and it meets AB at the midpoint M.

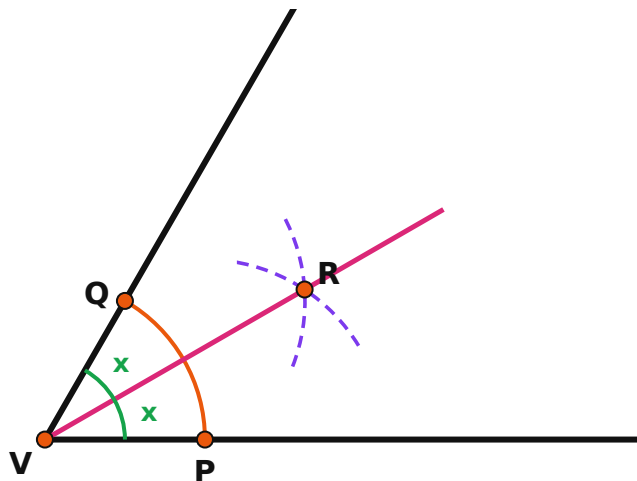


Perpendicular bisector: arcs of the same radius (more than half of AB) from A and from B.

This works because A, B, P and Q form a rhombus with all four sides equal to the compass radius, and the diagonals of a rhombus are perpendicular bisectors of each other.

How do you bisect an angle?

Draw an arc centered at the vertex V that crosses both rays at P and Q. From P and from Q draw arcs of equal radius that cross at R. Draw ray VR. Triangles VPR and VQR are congruent by SSS, so the two angles at V are equal.

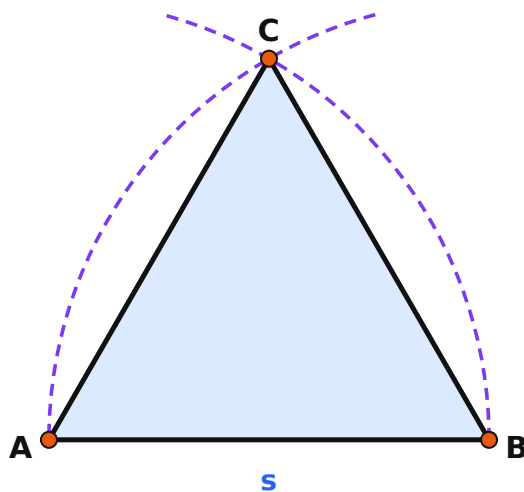


Angle bisector: arc across both rays, then two equal arcs meet at R.

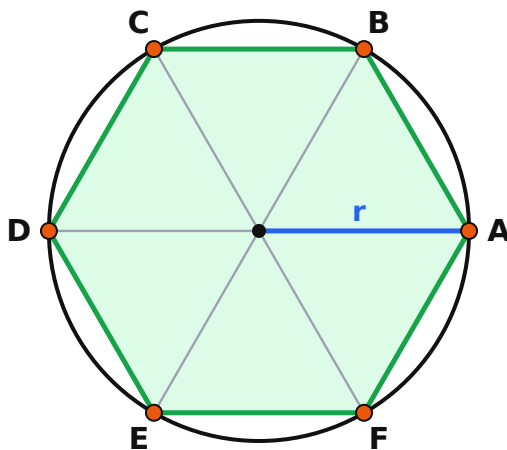
How do you construct an equilateral triangle, a square and a regular hexagon?

For an **equilateral triangle**, set the compass to AB and draw arcs from A and B. They meet at

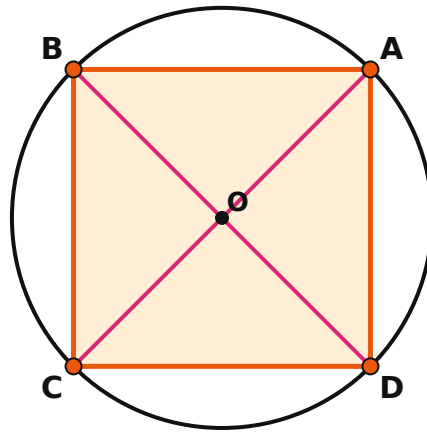
C, and $AC = BC = AB$. For a **regular hexagon**, draw a circle and, with the same radius, mark six points around it. For a **square**, draw a circle with two perpendicular diameters and join their four endpoints.



Equilateral triangle on AB: arcs of radius AB from A and from B meet at C.



A regular hexagon: the side equals the radius, so step the radius around the circle.



A square: draw two perpendicular diameters and join their endpoints.

Construction	Key idea
Copy a segment	compass set to the length
Copy an angle	copy the arc, then the chord
Parallel line	copy a corresponding angle
Perpendicular line	perpendicular bisector of a segment

Example: A perpendicular bisector is built from arcs of radius 9 on a segment AB of length 14. Then $AM = 7$ and $AP = 9$, so $PM = \sqrt{9^2 - 7^2} = \sqrt{32} = 4\sqrt{2}$.

Questions about geometric constructions

Why do constructions use only a compass and straightedge?

The tools have no measurements, so every step depends on equal radii and congruent triangles. This makes each construction a proof that the result is exact.

How do you copy an angle?

Draw an arc across the original angle, then draw the same arc on the new ray. Measure the chord between the crossing points with the compass and mark it on the new arc. Draw a ray through that point.

Why must the compass radius be more than half the segment when you bisect it?

If the radius is half of AB or less, the arcs from A and B do not cross. A larger radius makes them meet at two points.

How do you inscribe a regular hexagon in a circle?

Keep the compass at the circle's radius, mark a point on the circle, then step around the circle six times. Join the points in order.

How do you construct a perpendicular to a line through a point?

Mark two points on the line the same distance from the given point. Then construct the perpendicular bisector of the segment between them.