

Geometry - Coordinate Geometry and Proofs - Lesson

Common Core G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7 · maths-pdf.fr/en/

In the coordinate plane, parallel lines have equal slopes and perpendicular lines have slopes that are opposite reciprocals. A point that divides a segment in the ratio a to b is $a/(a + b)$ of the way from the first endpoint. The distance formula gives side lengths, perimeters and areas, and slope and distance tests prove properties of polygons.

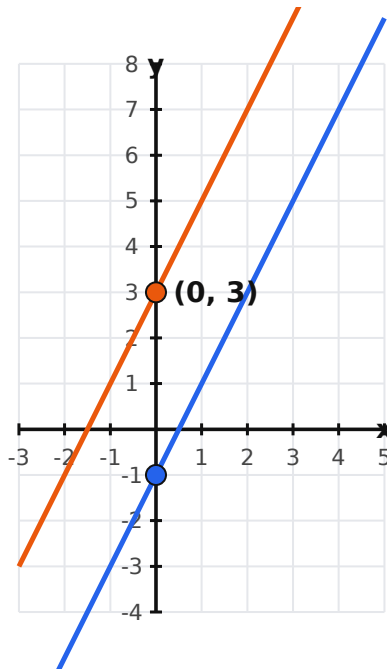
Geometry · Common Core State Standards: **G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7**

Key facts

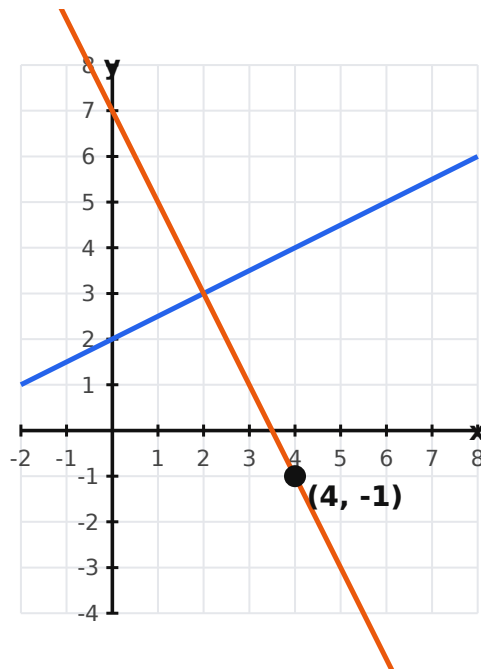
- Parallel lines: equal slopes. Perpendicular lines: slopes multiply to -1 .
- Point P with $AP:PB = a:b$ is $a/(a+b)$ of the way from A to B .
- Distance: square root of $(x_2 - x_1)^2$ plus $(y_2 - y_1)^2$ squared.
- Perimeter = sum of distances; area uses base times height or decomposition.
- Coordinate proofs test slopes (parallel, perpendicular) and distances (equal sides).

How do slopes show parallel and perpendicular lines?

The slope of the line through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$. In the coordinate plane, two nonvertical lines are **parallel** when their slopes are equal, and **perpendicular** when the product of their slopes is -1 , so the slopes are opposite reciprocals.



Parallel lines have the same slope.



Perpendicular slopes are opposite reciprocals.

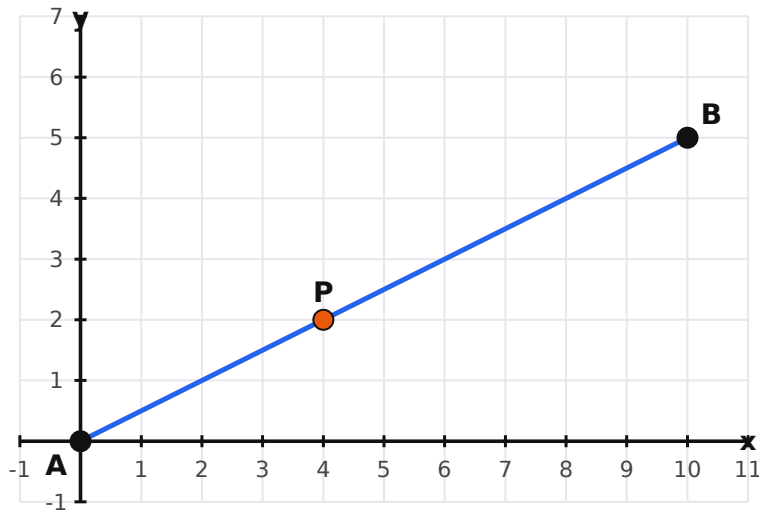
How do you write the equation of a parallel or perpendicular line?

1. Find the slope of the given line.
2. Use the same slope for a parallel line, or the opposite reciprocal for a perpendicular line.
3. Substitute the given point into $y - y_1 = m(x - x_1)$ and simplify.

How do you divide a segment in a given ratio?

To find the point P that splits segment AB so that $AP:PB = a:b$, move the fraction $\frac{a}{a+b}$ of the way from A to B:

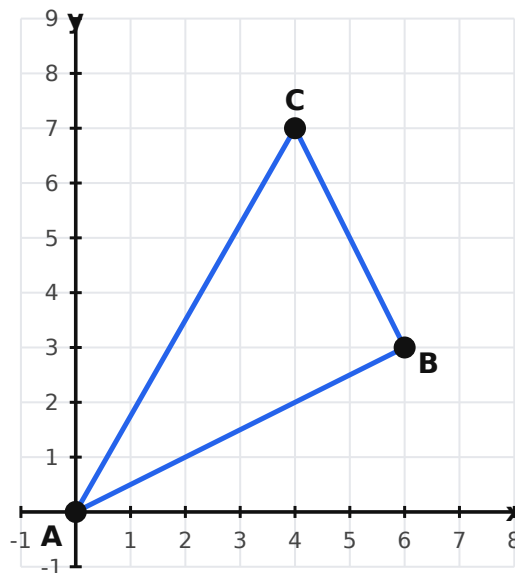
$$P = \left(x_1 + \frac{a}{a+b}(x_2 - x_1), y_1 + \frac{a}{a+b}(y_2 - y_1) \right)$$



P splits AB in the ratio 2 : 3 from A.

How do you find lengths, perimeter and area from coordinates?

The distance formula comes from the Pythagorean theorem: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Add the side lengths for the perimeter. For area, use a base and height when a side is horizontal or vertical, or split the polygon into rectangles and triangles.



Slope of AB = $1/2$; slope of AC = $7/4$.

How do you prove a fact with coordinates?

A **coordinate proof** places a figure on the plane and uses slope, distance and midpoint to test a property. To show that a quadrilateral is a parallelogram, show both pairs of opposite sides have equal slopes. To show a right angle, show two sides have slopes with product -1 . To show a triangle is isosceles, show two side lengths are equal.

Example: The point that is $\frac{2}{5}$ of the way from $A(0,0)$ to $B(10,5)$ is $(0+4, 0+2)=(4,2)$.

Questions about coordinate geometry and proofs

How do you know if two lines are perpendicular?

Multiply their slopes. If the product is -1 , the lines are perpendicular. Slopes $2/3$ and $-3/2$ multiply to -1 .

How do you find a point that divides a segment in a ratio?

Find the fraction $a/(a + b)$, then add that fraction of the x-change and of the y-change to the starting point.

What is the distance formula?

The distance between (x_1, y_1) and (x_2, y_2) is the square root of $(x_2 - x_1)$ squared plus $(y_2 - y_1)$ squared. It comes from the Pythagorean theorem.

How can you prove a quadrilateral is a parallelogram with coordinates?

Show that both pairs of opposite sides have equal slopes, or that the diagonals have the same midpoint.

How do you find the area of a triangle on the coordinate plane?

Use one half base times height when one side is horizontal or vertical. Otherwise enclose the triangle in a rectangle and subtract the extra triangles.