

Geometry - Cross Sections and Solids of Revolution - Lesson

Common Core G-GMD.B.4 · maths-pdf.fr/en/

A cross section is the shape made when a plane slices a solid. A cylinder cut parallel to its base gives a circle, a cut through its axis gives a rectangle, and every sphere section is a circle. Rotating a rectangle about a side makes a cylinder, and rotating a right triangle about a leg makes a cone.

Geometry · Common Core State Standards: **G-GMD.B.4**

Key facts

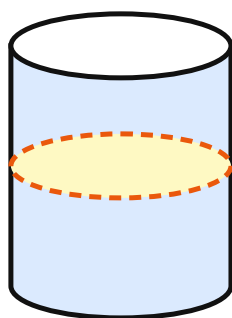
- Cylinder: parallel cut is a circle; cut through the axis is a rectangle.
- Cube: cut parallel to a face is a square.
- Cone: parallel cut is a circle; cut through the apex is a triangle.
- Sphere: every plane cross section is a circle.
- Rotating a rectangle about a side gives a cylinder; a right triangle about a leg gives a cone.

What is a cross section?

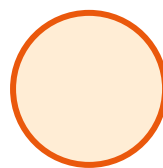
A **cross section** is the two-dimensional shape you see when a plane slices through a three-dimensional solid, like cutting a loaf of bread. The shape depends on the solid and on the angle of the cut.

What are the cross sections of a cylinder?

A plane parallel to the bases gives a circle. A plane perpendicular to the bases through the axis gives a rectangle. A slanted plane gives an ellipse.

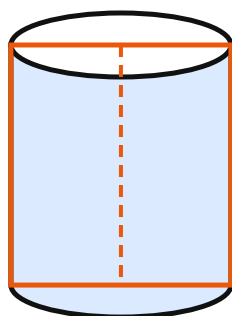


cross section

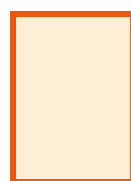


circle

A plane parallel to the base cuts a cylinder in a circle.

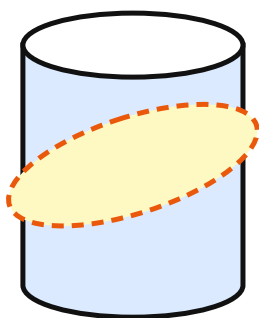


cross section

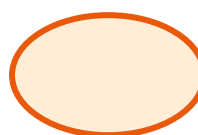


rectangle

A plane through the axis cuts a cylinder in a rectangle.



cross section

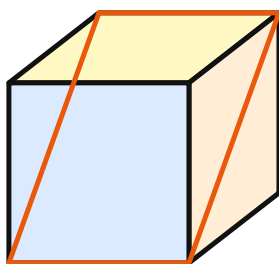


ellipse

A slanted plane cuts a cylinder in an ellipse.

What are the cross sections of a cube and a cone?

A plane parallel to a face of a cube gives a square. A plane through two opposite edges gives a rectangle, and a plane through three corners gives an equilateral triangle. For a cone, a plane parallel to the base gives a circle and a plane through the apex and axis gives a triangle.

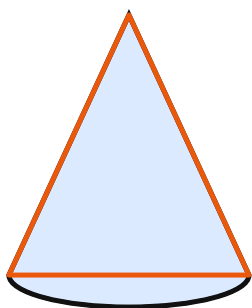


cross section

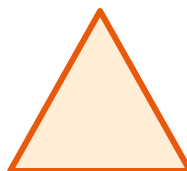


rectangle

A plane through opposite edges cuts a cube in a rectangle.



cross section



triangle

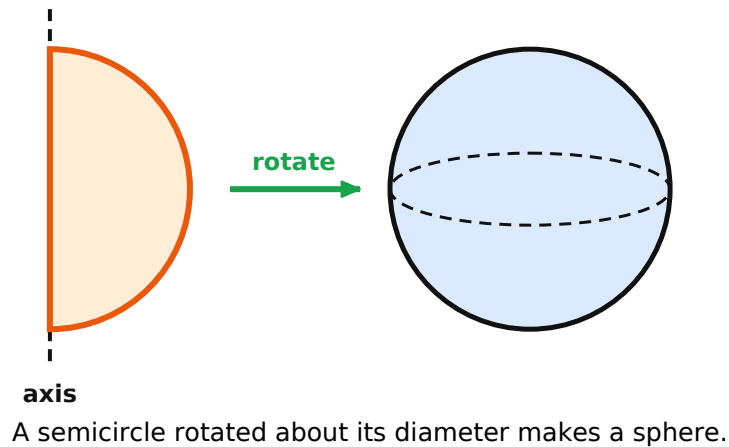
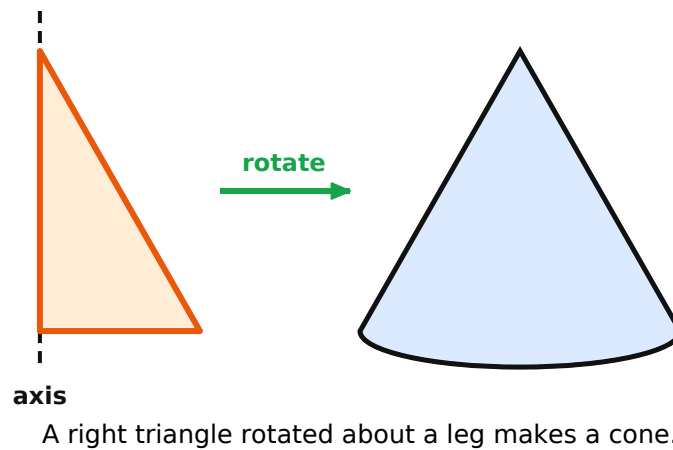
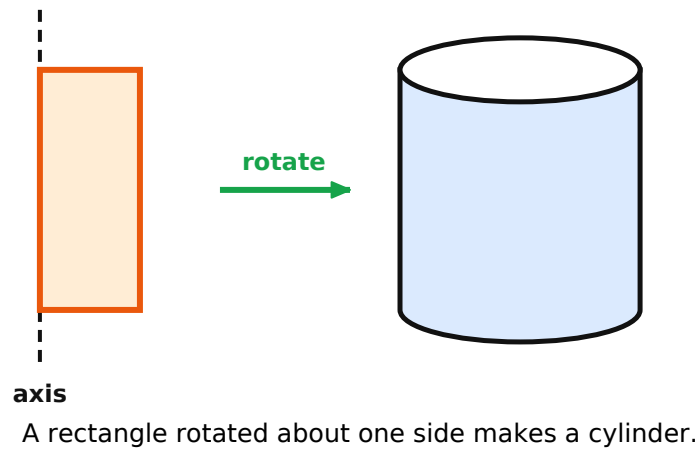
A plane through the apex and axis cuts a cone in a triangle.

Solid	Cut	Cross section
Cylinder	parallel to base	circle
Cylinder	through the axis	rectangle
Cube	parallel to a face	square
Cone	parallel to base	circle
Cone	through the apex	triangle
Sphere	any plane	circle

The cross section of a sphere is always a circle. A plane at distance d from the center of a sphere of radius r cuts a circle of radius $\sqrt{r^2 - d^2}$.

What is a solid of revolution?

When you rotate a two-dimensional shape about a line, the region it sweeps out is a **solid of revolution**. A rectangle rotated about one side makes a cylinder, a right triangle rotated about a leg makes a cone, and a semicircle rotated about its diameter makes a sphere.



1. Sketch the shape and the axis of rotation.
2. Decide which side lies on the axis: it becomes the height or the axis of the solid.
3. The distance from the axis becomes the radius.
4. Use the volume formula for the solid you found.

Example: A right triangle with legs 6 and 8 rotated about the leg of length 8 makes a cone with radius 6 and height 8, with volume $\frac{1}{3}\pi \cdot 6^2 \cdot 8 = 96\pi$.

Questions about cross sections and solids of revolution

What is a cross section in geometry?

A cross section is the plane figure formed where a plane cuts through a solid. For a cylinder cut parallel to its base it is a circle.

What cross sections can a cube have?

A cube can have a square, a rectangle, a triangle, a pentagon or a hexagon as a cross section, depending on how the plane cuts it.

What solid forms when a right triangle rotates about a leg?

A cone. The leg on the axis is the height and the other leg is the radius.

How do you find the radius of a circular cross section of a sphere?

If the plane is d units from the center of a sphere with radius r , the circle has radius the square root of $r^2 - d^2$.

What is a solid of revolution?

It is a three-dimensional shape formed by rotating a two-dimensional region about a line, such as a cylinder made from a rectangle.