

Geometry - Dilations and Similarity - Lesson

Common Core G-SRT.A.1, G-SRT.A.2, G-SRT.A.3 · maths-pdf.fr/en/

A dilation with center O and scale factor k multiplies every distance from O by k . It keeps angle measures, sends lines to parallel lines, and multiplies lengths by k and areas by k squared. Figures are similar when rigid motions and dilations map one onto the other. Triangles are similar if two pairs of angles are congruent (AA).

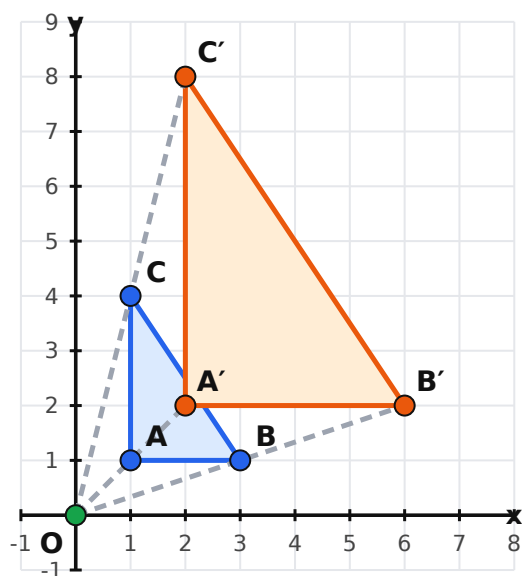
Geometry · Common Core State Standards: **G-SRT.A.1, G-SRT.A.2, G-SRT.A.3**

Key facts

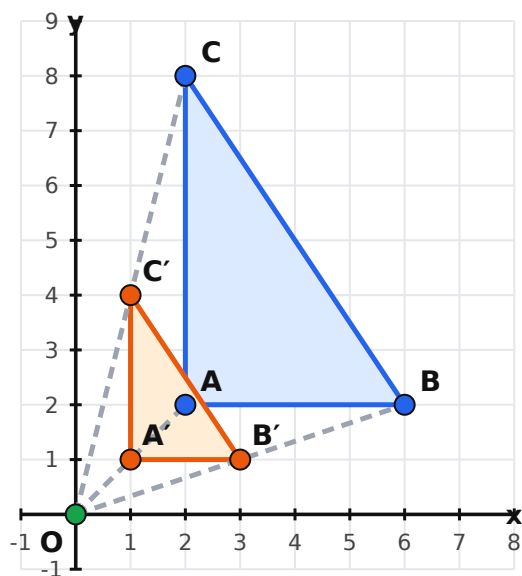
- A dilation multiplies lengths by the scale factor k ; with center at the origin, (x, y) goes to (kx, ky) .
- Dilations preserve angle measure and send lines to parallel lines.
- Area is multiplied by k squared and perimeter by k .
- A line through the center of dilation maps to itself.
- AA: two congruent angle pairs prove triangles similar.

What is a dilation?

A **dilation** with center O and scale factor k moves every point P to a point P' on ray OP so that $OP' = k \cdot OP$. If $k > 1$ the figure grows (an enlargement), and if $0 < k < 1$ it shrinks (a reduction). With the center at the origin, the rule is (x, y) maps to (kx, ky) .

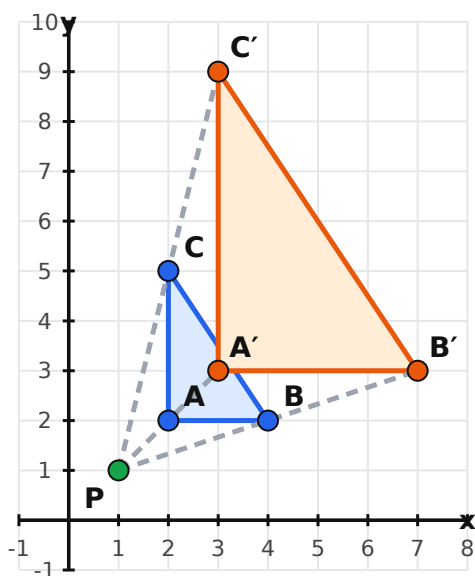


Dilation with center O and scale factor 2: every distance from O doubles.



Scale factor $1/2$ shrinks the figure toward the center.

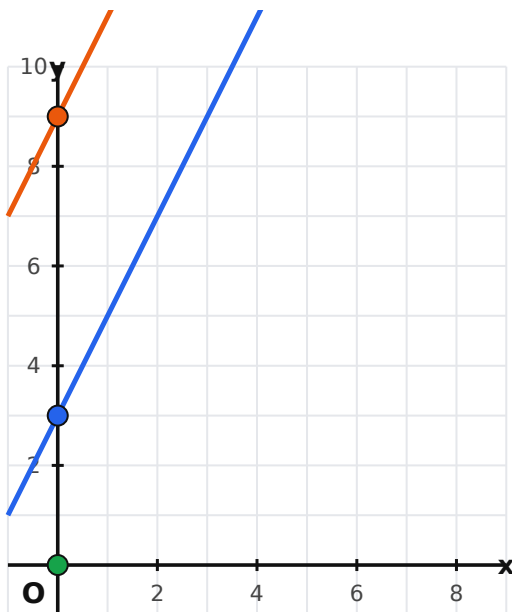
With another center $C(a,b)$, the image of P is $C + k(P - C)$. For example, dilating $(4,3)$ with center $(1,1)$ and $k = 2$ gives $(1 + 2 \cdot 3, 1 + 2 \cdot 2) = (7,5)$.



The center of dilation can be any point, here $P(1, 1)$.

What does a dilation change and what does it keep?

Changes	Stays the same
lengths are multiplied by k	angle measures
perimeter is multiplied by k	parallel lines stay parallel
area is multiplied by k^2	a line through the center maps to itself



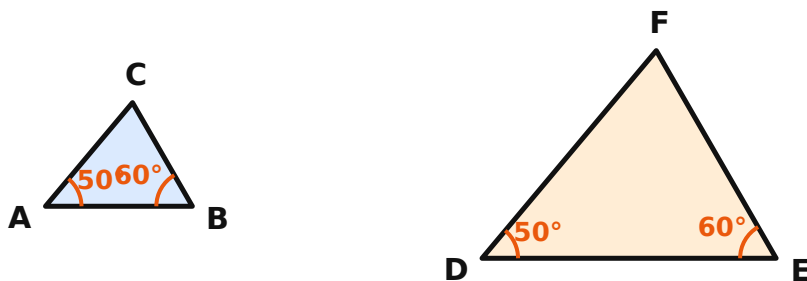
A line not through the center maps to a parallel line.

A line that does not pass through the center maps to a **parallel** line. A segment is multiplied

by exactly k .

When are two figures similar?

Two figures are **similar** when a sequence of rigid motions and dilations maps one onto the other. Corresponding angles are congruent and corresponding sides are proportional. For triangles, **AA** is enough: two pairs of congruent angles prove similarity, because the third angles must match too.



Similar triangles have equal angles and proportional sides.

How do you solve a similarity problem?

1. Match corresponding vertices in order.
2. Write the scale factor as a ratio of corresponding sides.
3. Set up a proportion for the unknown side.
4. Solve and check the answer is reasonable.

Example: Triangles ABC and DEF are similar with $AB = 6$, $DE = 9$ and $BC = 8$. The scale factor is $\frac{9}{6} = 1.5$, so $EF = 1.5 \cdot 8 = 12$.

Questions about dilations and similarity

What is the scale factor of a dilation?

It is the ratio of a length in the image to the corresponding length in the original figure. A scale factor of 3 triples every length, and $1/2$ halves it.

Is a dilation a rigid motion?

No, unless the scale factor is 1. A dilation changes lengths, so it does not preserve distance, although it does preserve angle measure.

How does a dilation affect area?

Area is multiplied by the square of the scale factor. If the scale factor is 3, the area becomes 9 times as large.

What does the AA similarity criterion say?

If two angles of one triangle are congruent to two angles of another, the triangles are similar. The third angles are then also congruent because angle sums are 180 degrees.

How do you dilate from a center that is not the origin?

Subtract the center from the point, multiply by the scale factor, and add the center back. The image of (4, 3) with center (1, 1) and $k = 2$ is (7, 5).