

Geometry - Law of Sines and Law of Cosines - Lesson

Common Core G-SRT.D.9, G-SRT.D.10, G-SRT.D.11 · maths-pdf.fr/en/

The Law of Sines says $a/\sin A = b/\sin B = c/\sin C$, and the Law of Cosines says $c^2 = a^2 + b^2 - 2ab \cos C$. Use sines for AAS and ASA, cosines for SAS and SSS. Area equals one half the product of two sides and the sine of the included angle.

Geometry · Common Core State Standards: **G-SRT.D.9, G-SRT.D.10, G-SRT.D.11**

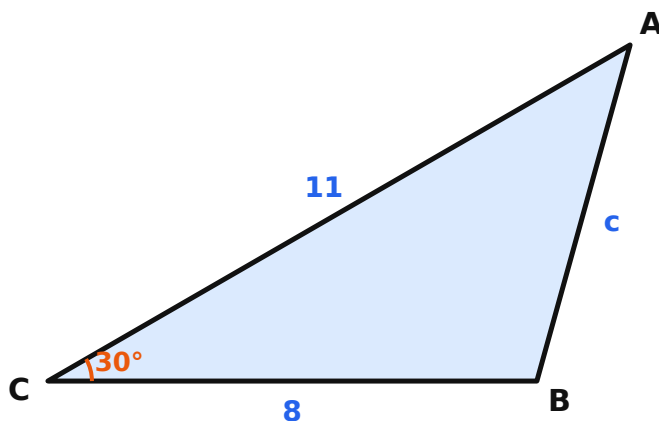
Key facts

- Area = $(1/2) ab \sin C$.
- Law of Sines: $a/\sin A = b/\sin B = c/\sin C$.
- Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.
- AAS or ASA: use sines. SAS or SSS: use cosines.
- When the included angle is 90 degrees the Law of Cosines becomes the Pythagorean theorem.

How do you find the area of a triangle with two sides and the included angle?

Drop an altitude from one vertex. The height is $b \sin C$, so the area is one half base times height:

$$\text{Area} = \frac{1}{2} ab \sin C$$



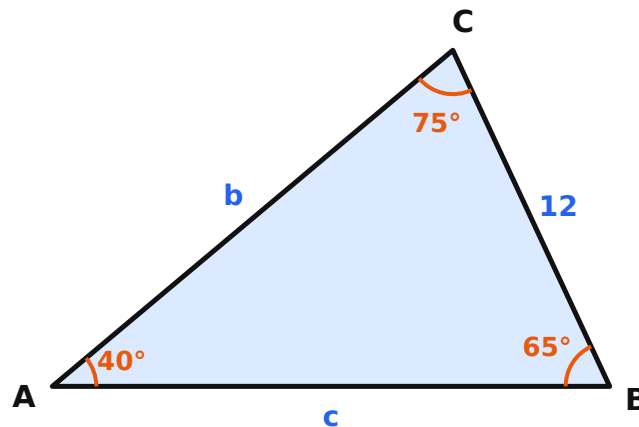
$$\text{Area} = (1/2)(8)(11) \sin 30^\circ = 22.$$

What is the Law of Sines?

In any triangle, each side divided by the sine of its opposite angle gives the same number.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Use it when you know two angles and a side (AAS or ASA), or two sides and an angle opposite one of them.



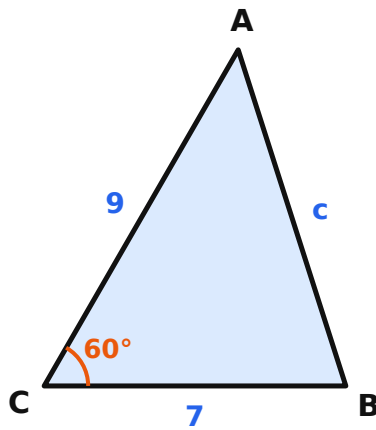
$$\text{Law of Sines: } a / \sin A = b / \sin B = c / \sin C.$$

What is the Law of Cosines?

The Law of Cosines extends the Pythagorean theorem to any triangle:

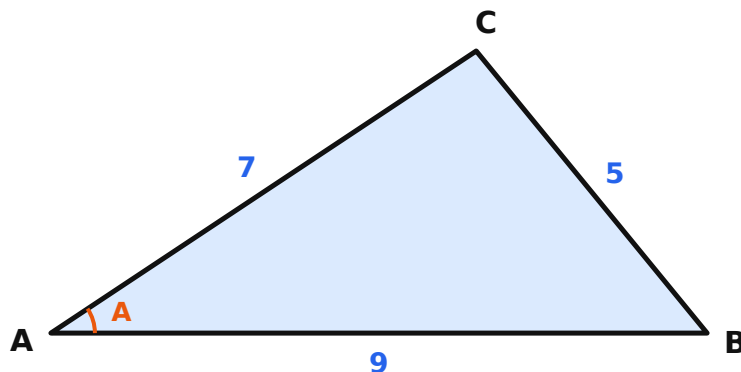
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Use it when you know two sides and the included angle (SAS) to find the third side, or all three sides (SSS) to find an angle. When $C = 90^\circ$, the cosine term is 0 and you get $c^2 = a^2 + b^2$.



Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$.

Given	First step
AAS or ASA	Law of Sines (find the third angle first)
SAS	Law of Cosines for the third side
SSS	Law of Cosines for an angle
SSA	Law of Sines (watch for two triangles)



Given three sides, use the Law of Cosines to find an angle.

How do you solve a non-right triangle?

1. Sketch the triangle and label the known parts.
2. Match the given pattern (AAS, SAS, SSS) to a law.
3. Substitute, solve, and round at the end.
4. Check that the angles add to 180° and the longest side faces the largest angle.

Example: Sides 7 and 9 have an included angle of 60° . Then $c^2 = 49 + 81 - 2 \cdot 7 \cdot 9 \cdot 0.5 = 67$, so $c = 8.2$.

Questions about law of sines and law of cosines

When do you use the Law of Sines?

Use it when you know two angles and a side, or two sides and an angle opposite one of them. You need a matching side and opposite angle pair.

When do you use the Law of Cosines?

Use it when you know two sides and the included angle (SAS) or all three sides (SSS). It also works for right triangles.

How do you find the area of a triangle without the height?

Use $\text{area} = (1/2) a b \sin C$ with two sides and their included angle. Two sides of 8 and 11 with a 30 degree angle give 22 square units.

Why does the Law of Cosines contain a cosine term?

The term corrects the Pythagorean theorem for the angle between the sides. At 90 degrees the cosine is 0 and the term disappears.

How do you check an answer to a triangle problem?

Make sure the angles add to 180 degrees and that the longest side is opposite the largest angle.