

Geometry - Law of Sines and Law of Cosines - Answer Key

Common Core G-SRT.D.9, G-SRT.D.10, G-SRT.D.11 · maths-pdf.fr/en/

Use this answer key to check the 20 problems on law of sines and cosines. Every solution shows the formula or theorem used, each step of the work, and the final answer in bold. If an answer differs from yours, compare the steps line by line to find where the work changed.

Geometry · Common Core State Standards: **G-SRT.D.9, G-SRT.D.10, G-SRT.D.11**

Exercise 1 - Area from SAS

$$\frac{1}{2} \cdot 8 \cdot 11 \cdot \sin 30^\circ = 44 \cdot 0.5 = 22$$

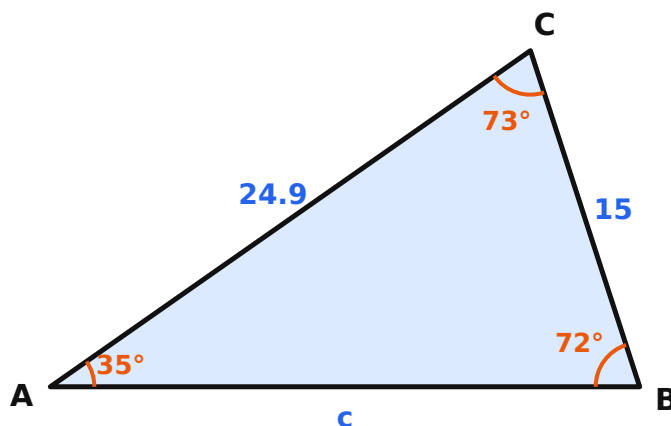
22 square units

Exercise 2 - Area of a sail

$$\frac{1}{2} \cdot 7.5 \cdot 12 \cdot \sin 58^\circ = 45 \cdot 0.8480 = 38.2$$

38.2 square meters

Exercise 3 - Law of Sines: find a side



b is about 24.9.

$$\frac{b}{\sin 72^\circ} = \frac{15}{\sin 35^\circ}$$

$$b = \frac{15 \cdot 0.9511}{0.5736} = 24.9$$

$$b = 24.9$$

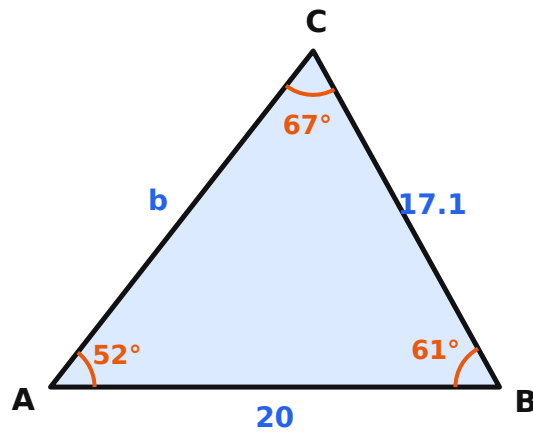
Exercise 4 - Law of Sines: find an angle

$$\sin B = \frac{15 \cdot \sin 50^\circ}{18} = 0.6384$$

$$B = \sin^{-1}(0.6384) = 39.7^\circ$$

$$B = 39.7^\circ$$

Exercise 5 - ASA



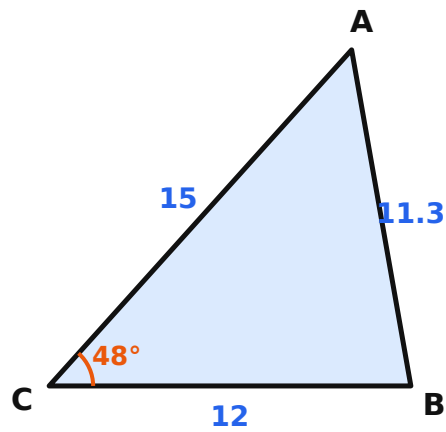
C = 67 degrees.

$$C = 180^\circ - 52^\circ - 61^\circ = 67^\circ.$$

$$\frac{a}{\sin 52^\circ} = \frac{20}{\sin 67^\circ}, \text{ so } a = 17.1.$$

$$C = 67^\circ; a = 17.1$$

Exercise 6 - Law of Cosines: SAS



c is about 11.3.

$$c^2 = 12^2 + 15^2 - 2 \cdot 12 \cdot 15 \cdot \cos 48^\circ$$

$$c^2 = 369 - 360 \cdot 0.6691 = 128.1$$

$$c = 11.3$$

$$\mathbf{c = 11.3}$$

Exercise 7 - Law of Cosines: SSS

$$5^2 = 7^2 + 9^2 - 2 \cdot 7 \cdot 9 \cdot \cos A$$

$$25 = 130 - 126 \cos A$$

$$\cos A = \frac{105}{126} = 0.8333$$

$$A = \cos^{-1}(0.8333) = 33.6^\circ$$

$$\mathbf{A = 33.6^\circ}$$

Exercise 8 - Obtuse angle

The largest angle is opposite 8.

$$8^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos C$$

$$\cos C = \frac{16 + 25 - 64}{40} = -0.575$$

$$C = \cos^{-1}(-0.575) = 125.1^\circ$$

$$\mathbf{125.1^\circ}$$

Exercise 9 - Right triangle check

$$25^2 = 7^2 + 24^2 - 2 \cdot 7 \cdot 24 \cdot \cos C$$

$$625 = 625 - 336 \cos C$$

$$\cos C = 0, \text{ so } C = 90^\circ.$$

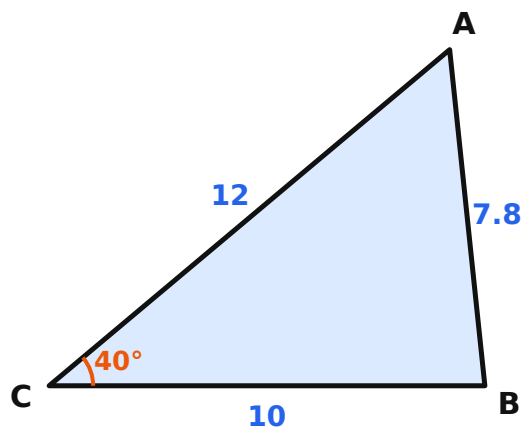
Right angle opposite the side 25

Exercise 10 - Which law?

(a) Law of Sines. (b) Law of Cosines. (c) Law of Cosines. (d) Law of Sines.

a: sines; b: cosines; c: cosines; d: sines

Exercise 11 - Solve a triangle (SAS)



Triangle ABC with c about 7.8.

$$c^2 = 100 + 144 - 240 \cos 40^\circ = 60.1, \text{ so } c = 7.8.$$

$$\sin A = \frac{10 \sin 40^\circ}{7.8} = 0.8288, \text{ so } A = 56.0^\circ.$$

$$B = 180^\circ - 40^\circ - 56.0^\circ = 84.0^\circ.$$

$$\mathbf{c = 7.8; A = 56.0^\circ; B = 84.0^\circ}$$

Exercise 12 - Angle opposite the shorter side

$$\sin B = \frac{18 \cdot \sin 58^\circ}{22} = 0.6939$$

$$B = \sin^{-1}(0.6938) = 43.9^\circ$$

$$\mathbf{B = 43.9^\circ}$$

Exercise 13 - Equilateral triangle area

$$\frac{1}{2} \cdot 10 \cdot 10 \cdot \sin 60^\circ = 50 \cdot \frac{\sqrt{3}}{2} = 25\sqrt{3}.$$

$$\mathbf{25\sqrt{3}, \text{ about } 43.3}$$

Exercise 14 - Angle from area

$$30 = \frac{1}{2} \cdot 10 \cdot 12 \cdot \sin C$$

$$\sin C = \frac{30}{60} = 0.5$$

$$C = 30^\circ$$

$$\mathbf{30^\circ}$$

Exercise 15 - Across a lake

$$AB^2 = 420^2 + 380^2 - 2 \cdot 420 \cdot 380 \cdot \cos 48^\circ$$

$$AB^2 = 320,800 - 319,200 \cdot 0.6691 = 107,214$$

$$AB = 327$$

$$\mathbf{\text{About } 327 \text{ feet}}$$

Exercise 16 - Ship's return

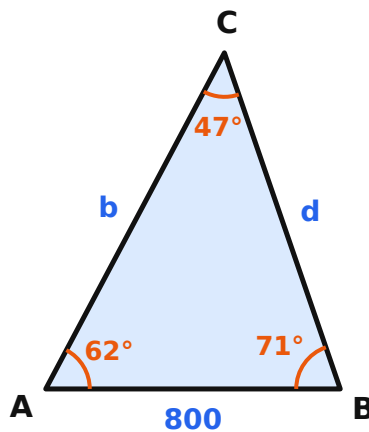
$$d^2 = 40^2 + 55^2 - 2 \cdot 40 \cdot 55 \cdot \cos 110^\circ$$

$$d^2 = 4625 + 4400 \cdot 0.3420 = 6,130$$

$$d = 78$$

$$\mathbf{\text{About } 78 \text{ miles}}$$

Exercise 17 - Fire lookouts



Lookouts at A and B, fire at C.

Angle at the fire: $180^\circ - 62^\circ - 71^\circ = 47^\circ$.

The side opposite the 71° angle is the distance from the 62° lookout.

$$\frac{d}{\sin 71^\circ} = \frac{800}{\sin 47^\circ}, \text{ so } d = 1034$$

About 1034 feet

Exercise 18 - Hikers apart

$$d^2 = 5.4^2 + 3.8^2 - 2 \cdot 5.4 \cdot 3.8 \cdot \cos 112^\circ$$

$$d^2 = 43.6 + 41.04 \cdot 0.3746 = 59.0$$

$$d = 7.7$$

About 7.7 km

Exercise 19 - Garden lot cost

$$\text{Area: } \frac{1}{2} \cdot 120 \cdot 150 \cdot \sin 35^\circ = 9000 \cdot 0.5736 = 5,162 \text{ square feet.}$$

$$\text{Cost: } \$0.45 \cdot 5,162 = \$2,323.$$

Area about 5,162 sq ft; cost about \$2,323

Exercise 20 - Order the sides

The longest side is opposite the largest angle, 80° .

$$\frac{b}{\sin 65^\circ} = \frac{6}{\sin 35^\circ}, \text{ so } b = 9.5.$$

$$c = \frac{6 \cdot \sin 80^\circ}{\sin 35^\circ} = 10.3.$$

Longest side $c = 10.3$; $b = 9.5$