

Geometry - Parallelograms and Quadrilaterals - Lesson

Common Core G-CO.C.11 · maths-pdf.fr/en/

A parallelogram is a quadrilateral with two pairs of parallel sides. Its opposite sides and opposite angles are congruent, consecutive angles are supplementary, and its diagonals bisect each other. A rectangle also has congruent diagonals, and a rhombus has perpendicular diagonals. These properties let you write equations for unknown lengths and angles.

Geometry · Common Core State Standards: **G-CO.C.11**

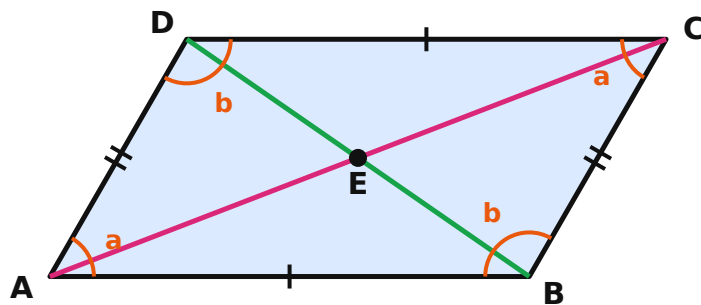
Key facts

- Parallelogram: opposite sides congruent, opposite angles congruent, diagonals bisect each other.
- Consecutive angles of a parallelogram add to 180 degrees.
- Rectangle: four right angles and congruent diagonals.
- Rhombus: four congruent sides and perpendicular diagonals that bisect the angles.
- The angles of any quadrilateral add to 360 degrees.

What is a parallelogram and what are its properties?

A **parallelogram** is a quadrilateral with two pairs of parallel sides. Because of the parallel sides, every parallelogram has these properties.

- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary (they add to 180°).
- The diagonals bisect each other.

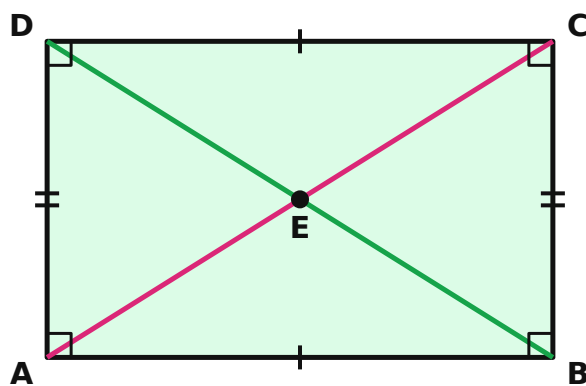


A parallelogram: opposite sides and opposite angles are congruent.

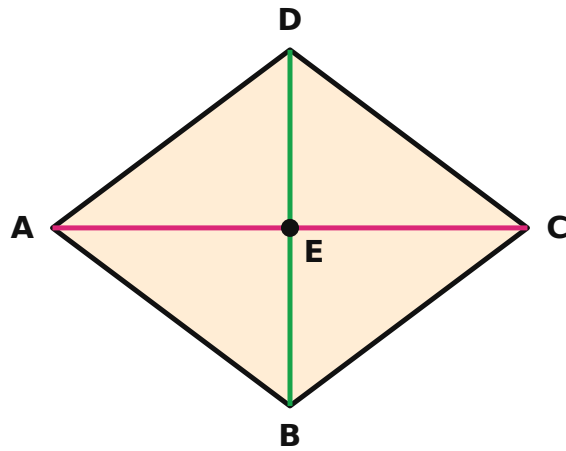
Each property can be proved by splitting the parallelogram along a diagonal. The two triangles are congruent by ASA, and CPCTC gives the congruent sides and angles.

What extra properties do special parallelograms have?

Shape	Extra properties
Rectangle	four right angles; diagonals are congruent
Rhombus	four congruent sides; diagonals are perpendicular and bisect the angles
Square	all of the above



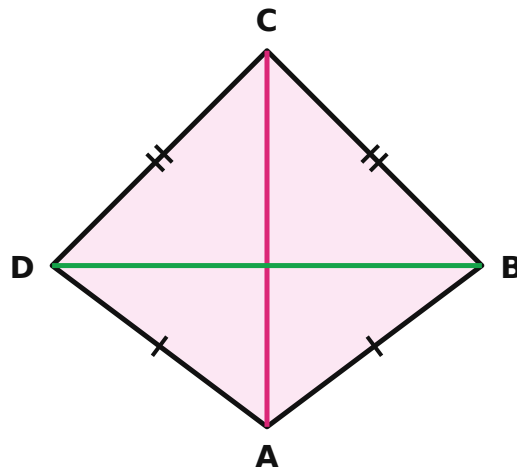
A rectangle has congruent diagonals.



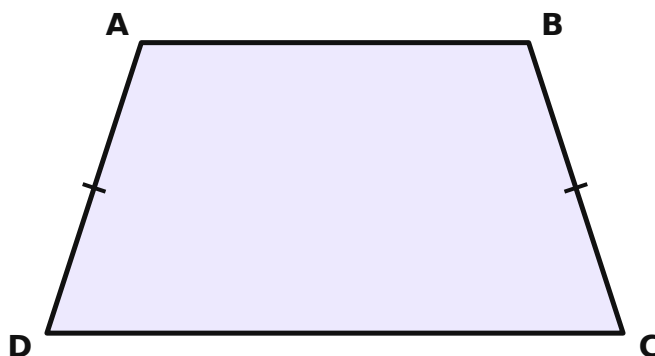
A rhombus has perpendicular diagonals that bisect its angles.

What about kites and trapezoids?

A **kite** has two pairs of adjacent congruent sides. Its diagonals are perpendicular, and one diagonal bisects the other. A **trapezoid** has exactly one pair of parallel sides, called the bases. In an isosceles trapezoid the legs are congruent and each pair of base angles is congruent.



A kite has two pairs of adjacent congruent sides.



An isosceles trapezoid has congruent legs and congruent base angles.

These shapes are nested. Every square is a rhombus and a rectangle, every rectangle and rhombus is a parallelogram, and every parallelogram is a quadrilateral. A property of a bigger family always holds for the smaller ones inside it. For example, a square has all the properties of a rectangle, so its diagonals are congruent, and all the properties of a rhombus, so its diagonals are perpendicular.

How do you find missing measures in a quadrilateral?

1. Identify the quadrilateral and list its properties.
2. Choose the property that links the given and unknown measures.
3. Write an equation and solve it.
4. Check with a second property, such as the angle sum of 360° .

Example: In parallelogram ABCD the diagonals meet at E with $AE = 3x + 1$ and $EC = 5x - 9$. The diagonals bisect each other, so $3x + 1 = 5x - 9$, giving $x = 5$. Then $AE = 16$ and $AC = 32$.

Questions about parallelograms and quadrilaterals

How do you prove a quadrilateral is a parallelogram?

Show that both pairs of opposite sides are parallel or congruent, or that the diagonals bisect each other. One pair of sides that is both parallel and congruent also works.

Is a square a rectangle?

Yes. A square has four right angles and congruent diagonals, so it is a rectangle. It also has four congruent sides, so it is a rhombus too.

What is true about the diagonals of a rhombus?

They are perpendicular and they bisect each other and the angles of the rhombus. A rhombus

with diagonals 16 and 30 has side length 17.

Are the angles of a parallelogram all equal?

No. Opposite angles are equal, and consecutive angles add to 180 degrees, unless the parallelogram is a rectangle.

What makes a trapezoid isosceles?

An isosceles trapezoid has congruent legs. Its base angles are congruent in pairs, and its diagonals are congruent.