

# Geometry - Similar Triangle Theorems - Lesson

Common Core G-SRT.B.4, G-SRT.B.5 · maths-pdf.fr/en/

**A line parallel to one side of a triangle divides the other two sides proportionally and cuts off a similar triangle. Triangles are similar by AA, SAS~ or SSS~. In a right triangle the altitude to the hypotenuse makes two similar triangles, which gives the geometric mean relationships and a proof of the Pythagorean theorem.**

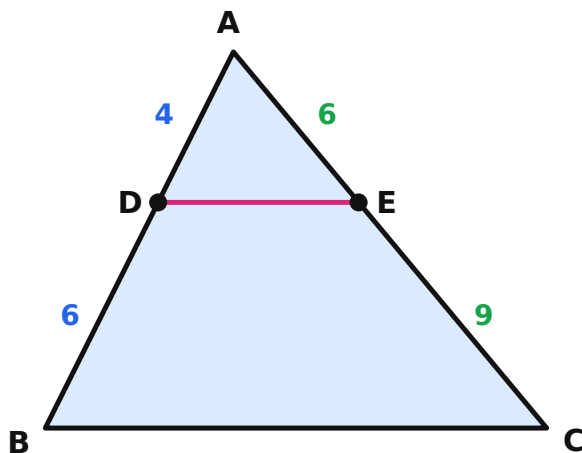
**Geometry** · Common Core State Standards: **G-SRT.B.4, G-SRT.B.5**

## Key facts

- Triangle Proportionality: DE parallel to BC gives  $AD/DB = AE/EC$ .
- Triangle similarity tests: AA, SAS~ and SSS~.
- Altitude to the hypotenuse:  $h^2 = p \times q$ .
- Leg relationship:  $a^2 = p \times c$ .
- Similar triangles let you measure heights and widths indirectly.

## What does a line parallel to a side do to a triangle?

The **Triangle Proportionality Theorem** says that a line parallel to one side of a triangle divides the other two sides proportionally. In the figure,  $DE$  is parallel to  $BC$ , so  $\frac{AD}{DB} = \frac{AE}{EC}$ .

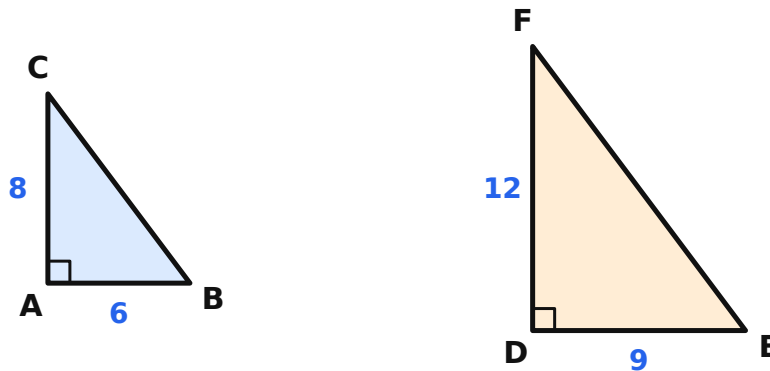


A line parallel to BC splits both sides in the same ratio.

The converse is also true. If a line divides two sides in the same ratio, it is parallel to the third side. Triangle ADE is also similar to triangle ABC by AA, because the parallel line creates congruent corresponding angles.

## How can you show triangles are similar?

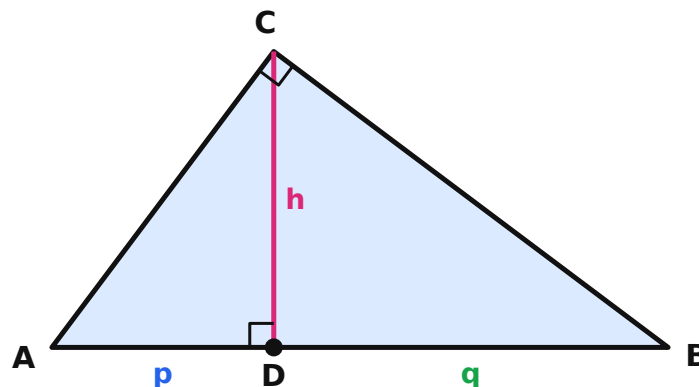
| Test | What you check                                                    |
|------|-------------------------------------------------------------------|
| AA   | two pairs of congruent angles                                     |
| SAS~ | two pairs of proportional sides and the included angles congruent |
| SSS~ | three pairs of proportional sides                                 |



SAS similarity: the included angles are equal and the sides are proportional ( $6/9 = 8/12$ ).

## What happens in a right triangle with an altitude to the hypotenuse?

The altitude from the right angle to the hypotenuse splits the triangle into two smaller triangles. Both are similar to the original triangle and to each other. This gives the geometric-mean relationships.



The altitude CD to the hypotenuse makes two triangles similar to ABC.

### Geometric mean relationships

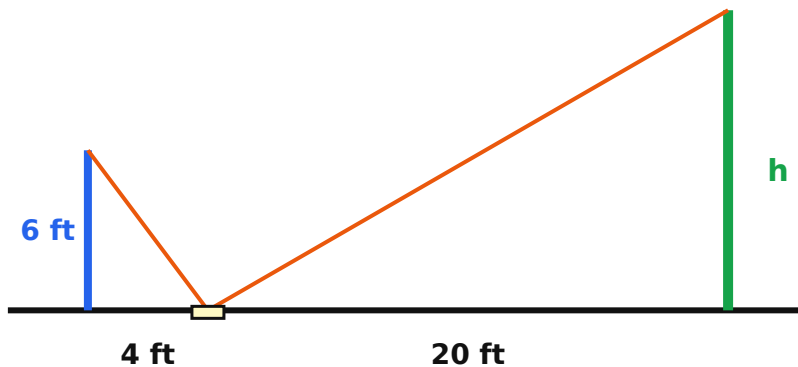
The altitude is the geometric mean of the two segments of the hypotenuse:  $h^2 = pq$ . Each leg is the geometric mean of the hypotenuse and the segment next to it:  $a^2 = pc$  and  $b^2 = qc$ .

## How does similarity prove the Pythagorean theorem?

Add the two leg relationships:  $a^2 + b^2 = pc + qc = (p+q)c = c \cdot c = c^2$ . This is the Pythagorean theorem, proved with similar triangles.

## How do you measure something you cannot reach?

Similar triangles let you find heights and widths indirectly, with a shadow or a mirror.



Reflection method: two similar right triangles.

1. Draw the two triangles and mark the equal angles.
2. Write a proportion of corresponding sides.
3. Solve for the unknown length.

**Example:** In the mirror figure,  $\frac{h}{20} = \frac{6}{4}$ , so  $h = 30$  feet.

## Questions about similar triangle theorems

### What is the Triangle Proportionality Theorem?

If a line is parallel to one side of a triangle and meets the other two sides, it divides them proportionally. For example, if  $AD = 4$ ,  $DB = 6$  and  $AE = 6$ , then  $EC = 9$ .

### How can you tell whether a line is parallel to a side?

Check the converse: if the line divides two sides in equal ratios, it is parallel to the third side. If the ratios differ, it is not parallel.

### What is a geometric mean?

The geometric mean of two positive numbers  $a$  and  $b$  is the square root of their product. The

geometric mean of 4 and 9 is 6.

### **How does similarity prove the Pythagorean theorem?**

The altitude to the hypotenuse gives  $a^2 = pc$  and  $b^2 = qc$ . Adding the equations gives  $a^2 + b^2 = (p + q)c = c^2$ .

### **What is the difference between SAS congruence and SAS similarity?**

SAS congruence needs two pairs of equal sides and the included angle. SAS similarity needs two pairs of proportional sides and the included angle equal.