

Geometry - Symmetry of Polygons and Rigid Motions - Lesson

Common Core G-CO.A.3, G-CO.B.6 · maths-pdf.fr/en/

A figure has **line symmetry** when a reflection carries it onto itself, and **rotational symmetry** when a turn of less than 360 degrees does. A regular polygon with n sides has n lines of symmetry and rotates onto itself every 360 divided by n degrees. Reflections and rotations are rigid motions, so they preserve distance and angle measure.

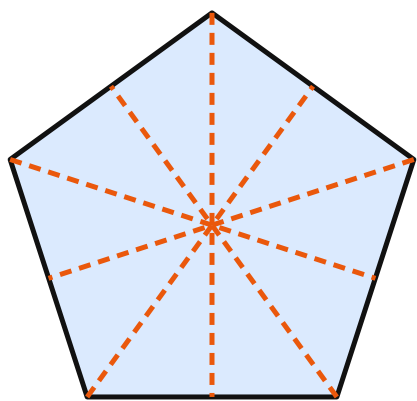
Geometry · Common Core State Standards: **G-CO.A.3, G-CO.B.6**

Key facts

- A regular n -gon has n lines of symmetry and rotational symmetry every $360/n$ degrees.
- A rectangle that is not a square has 2 lines of symmetry and a 180 degree rotation.
- A parallelogram that is not a rectangle or rhombus has no line of symmetry but has a 180 degree rotation.
- Rigid motions preserve distance and angle measure.
- To test a symmetry, check that every vertex maps to a vertex.

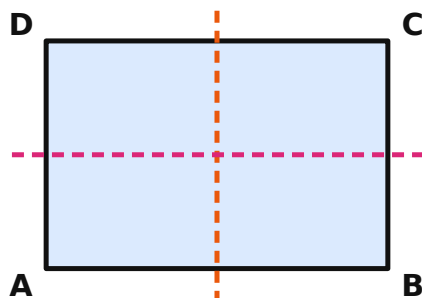
What is line symmetry?

A figure has **line symmetry** when a reflection across some line carries the figure onto itself. That line is a **line of symmetry**. Every vertex lands on a vertex, and every side lands on a side of the same length.



A regular pentagon has 5 lines of symmetry.

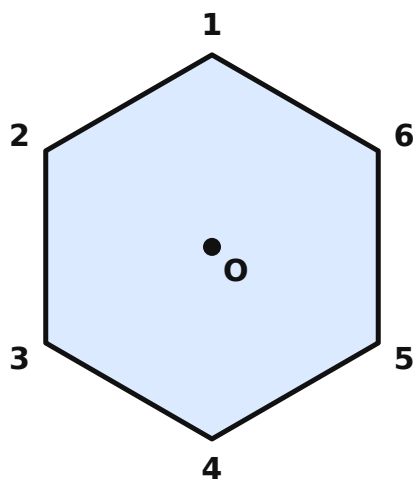
A regular polygon with n sides has exactly n lines of symmetry. A rectangle that is not a square has 2, a square has 4, and an isosceles trapezoid has 1.



A rectangle has 2 lines of symmetry.

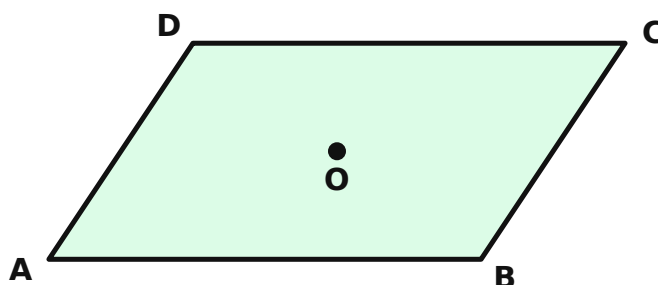
What is rotational symmetry?

A figure has **rotational symmetry** when a rotation of less than 360° about its center carries it onto itself. A regular polygon with n sides maps onto itself for rotations of $\frac{360^\circ}{n}$ and every multiple of that angle.



A regular hexagon turns onto itself every 60 degrees.

Figure	Lines of symmetry	Smallest rotation
Equilateral triangle	3	120°
Square	4	90°
Rectangle	2	180°
Parallelogram	0	180°
Regular hexagon	6	60°



A parallelogram has rotational symmetry about O, but no line of symmetry.

Why do rigid motions matter here?

Reflections and rotations are **rigid motions**. They preserve the distance between points and the measure of every angle. That is why a symmetry carries sides onto sides of equal length and angles onto angles of equal measure. In general, two figures are congruent when a rigid motion carries one onto the other.

Test for a symmetry

To check a symmetry, write where each vertex goes. If the images are exactly the original vertices, the motion carries the figure onto itself.

How do you find the symmetries of a polygon?

1. Sketch the polygon and mark its center.
2. Look for lines that split it into two mirror images and count them.
3. Find the smallest turn about the center that matches the figure, using $360^\circ \div n$ for a regular polygon.
4. Verify with vertices: each vertex must map to a vertex.

Example: A regular nonagon has 9 sides. It has 9 lines of symmetry, and its smallest rotation is $360^\circ \div 9 = 40^\circ$. The rotations that map it onto itself are 40° , 80° , 120° and so on up to 360° .

Questions about symmetry of polygons and rigid motions

How many lines of symmetry does a square have?

A square has 4 lines of symmetry: two through opposite vertices and two through the midpoints of opposite sides. It also has rotational symmetry of 90, 180 and 270 degrees.

Does a parallelogram have line symmetry?

A general parallelogram has no line of symmetry. It has rotational symmetry of 180 degrees about the point where its diagonals meet.

How do you find the angle of rotational symmetry of a regular polygon?

Divide 360 by the number of sides. A regular decagon has 10 sides, so its smallest rotation is 36 degrees.

What is the order of rotational symmetry?

The order is the number of times a figure matches itself during one full turn. A regular hexagon has order 6.

What is a rigid motion?

A rigid motion is a transformation that preserves distance and angle measure: a translation, a reflection, a rotation, or a combination of them.