

Geometry - Translations, Reflections and Rotations - Lesson

Common Core G-CO.A.2, G-CO.A.4, G-CO.A.5 · maths-pdf.fr/en/

A transformation maps each point of a figure to a new point. Translations slide a figure, reflections flip it across a line, and rotations turn it about a center. These three rigid motions keep lengths and angle measures the same. Each has a coordinate rule.

Geometry · Common Core State Standards: **G-CO.A.2, G-CO.A.4, G-CO.A.5**

Key facts

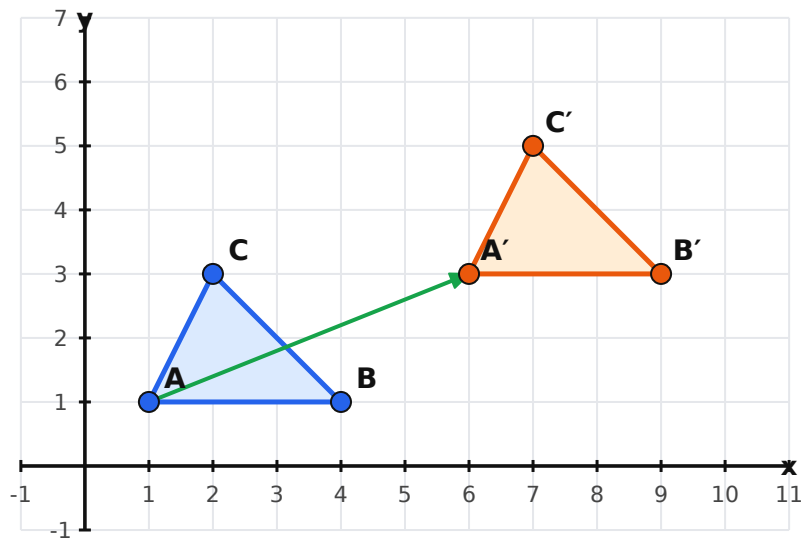
- A translation moves every point the same distance in the same direction: (x, y) to $(x + a, y + b)$.
- A reflection across the x-axis sends (x, y) to $(x, -y)$; across the y-axis it sends (x, y) to $(-x, y)$.
- A 90 degree counterclockwise rotation about the origin sends (x, y) to $(-y, x)$.
- Rigid motions preserve distance and angle measure, so the image is congruent to the preimage.
- When transformations are combined, the order matters.

What is a transformation?

A **transformation** takes each point of a figure, called the preimage, and maps it to a new point. The new figure is the **image**. We name image points with a prime mark, so A becomes A'. A transformation that keeps all distances and angle measures the same is a **rigid motion**.

How does a translation work?

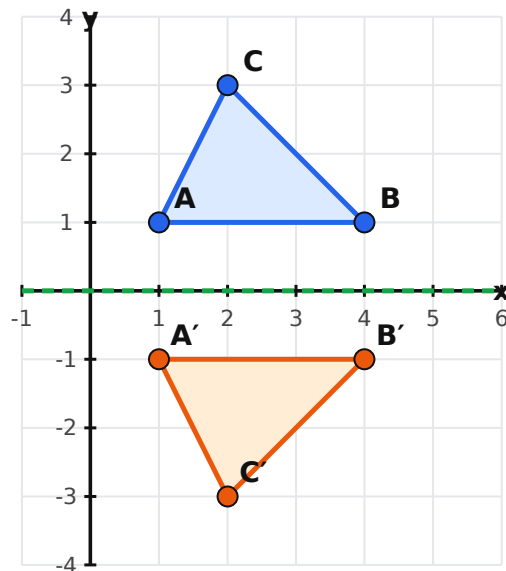
A **translation** slides every point the same distance in the same direction. On the coordinate plane, the rule is (x, y) maps to $(x + a, y + b)$.



Translation: every point moves 5 right and 2 up.

How does a reflection work?

A **reflection** flips a figure across a line of reflection. The line is the perpendicular bisector of every segment that joins a point to its image.



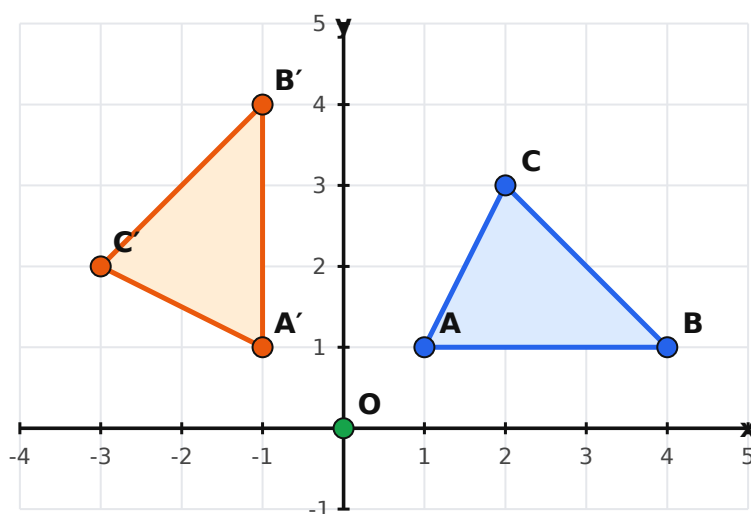
Reflection across the x-axis: (x, y) goes to $(x, -y)$.

Reflection across	Rule
the x-axis	(x, y) maps to $(x, -y)$
the y-axis	(x, y) maps to $(-x, y)$

Reflection across	Rule
the line $y = x$	(x,y) maps to (y,x)

How does a rotation work?

A **rotation** turns a figure about a fixed point, the center, by a given angle. A positive angle means counterclockwise. Every point stays the same distance from the center.

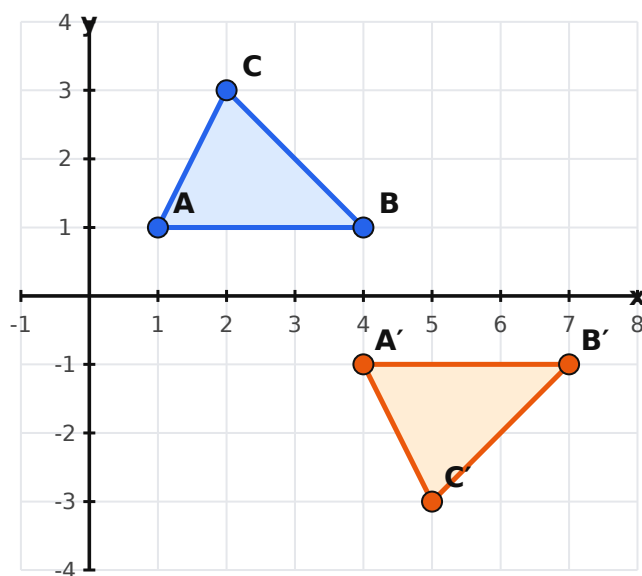


Rotation of 90° counterclockwise about the origin: (x, y) goes to $(-y, x)$.

Rotation about the origin	Rule
90° counterclockwise	(x,y) maps to $(-y,x)$
180°	(x,y) maps to $(-x,-y)$
270° counterclockwise	(x,y) maps to $(y,-x)$

How do you combine transformations?

You can apply one transformation to the image of another. The order can change the result, so always work in the order given.



A reflection followed by a translation is a sequence of rigid motions.

Example: Rotate $P(3,-2)$ by 90° counterclockwise about the origin, then translate by $(x+4,y)$. The rotation gives $(2,3)$, and the translation gives $(6,3)$.

Questions about translations, reflections and rotations

What is the difference between a preimage and an image?

The preimage is the original figure and the image is the figure after the transformation. Image points are named with a prime, so the image of A is A prime.

Which transformations are rigid motions?

Translations, reflections and rotations are rigid motions because they preserve distance and angle measure. A dilation is not a rigid motion because it changes lengths.

How do you reflect a point across the y-axis?

Change the sign of the x-coordinate and keep the y-coordinate. The point $(4, 6)$ reflects to $(-4, 6)$.

Which way is a positive rotation?

Counterclockwise is positive. A rotation of 90 degrees clockwise is the same as a rotation of 270 degrees counterclockwise.

Does the order of two transformations matter?

Often yes. Translating $(3, 4)$ by $(x + 2, y - 1)$ and then reflecting across the x -axis gives $(5, -3)$, but reflecting first gives $(5, -5)$.