

# Grade 8 - Dilations and Similar Figures - Lesson

Common Core 8.G.A.3, 8.G.A.4 · maths-pdf.fr/en/

**A dilation enlarges or reduces a figure by a scale factor  $k$  from a center point; from the origin, each coordinate is multiplied by  $k$ . The image has equal angles but lengths multiplied by  $k$ , so it is similar. Two figures are similar if a sequence of rotations, reflections, translations and dilations maps one onto the other.**

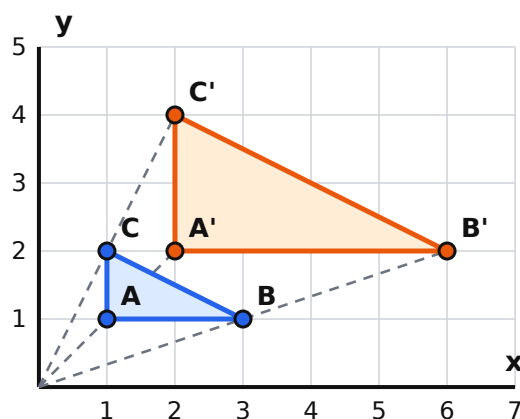
**Grade 8** · Common Core State Standards: **8.G.A.3, 8.G.A.4**

## Key facts

- Dilation from the origin:  $(x,y) \rightarrow (kx,ky)$ .
- $k > 1$  enlarges;  $0 < k < 1$  reduces;  $k = 1$  leaves the figure unchanged.
- Angles stay equal; side lengths multiply by  $k$ ; area multiplies by  $k^2$ .
- Similar figures have equal angles and proportional sides.
- Use  $\frac{\text{new}}{\text{old}} = k$  to find missing lengths.

## What is a dilation?

A dilation resizes a figure from a fixed point called the center. The scale factor  $k$  tells how much the figure grows or shrinks. If  $k > 1$ , it is an enlargement; if  $0 < k < 1$ , it is a reduction. When the center is the origin,  $(x,y) \rightarrow (kx,ky)$ .

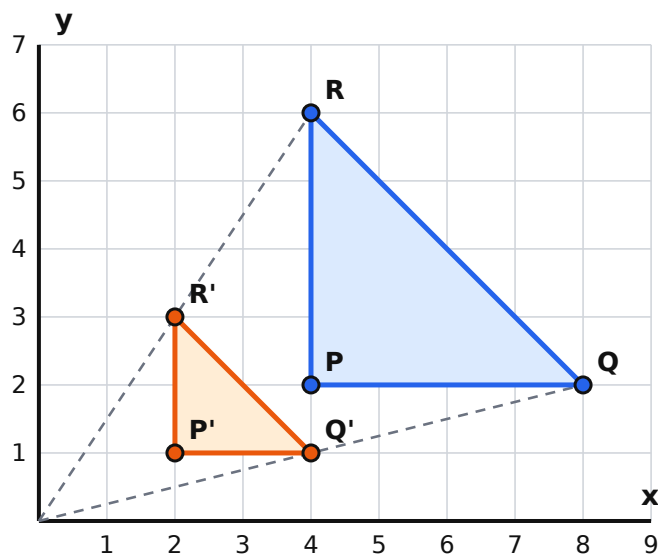


Scale factor 2 from the origin.

## Dilation facts

A dilation keeps **angle measures** and **parallel lines** but multiplies every length by  $k$ . Perimeters scale by  $k$  and areas scale by  $k^2$ . The image is **similar** to the original, not

congruent unless  $k = 1$ .

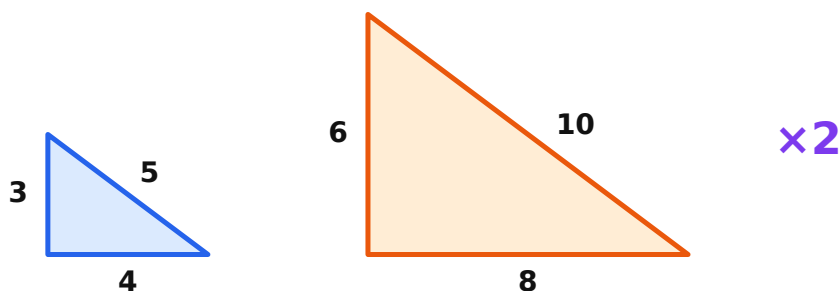


Scale factor one half shrinks the figure.

To check a dilation, draw a dashed ray from the center through each vertex. The image vertex lies on that ray, at  $k$  times the distance from the center. For the triangle with vertex  $(4,2)$  and scale factor  $\frac{1}{2}$ , the image vertex is at  $(2,1)$ , halfway from the origin. Reductions bring the figure closer to the center and enlargements push it farther away. The center itself never moves.

### What are similar figures?

Two figures are **similar** if one can be mapped onto the other by a sequence of rotations, reflections, translations and dilations. Matching angles are equal and matching sides are proportional.



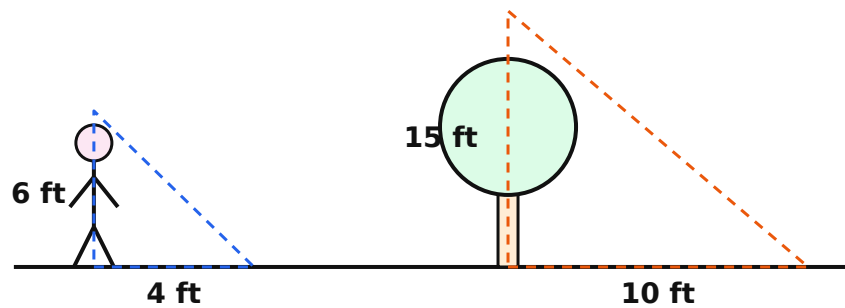
Each side of the large triangle is 2 times the matching side.

The triangle with sides 3, 4, 5 and the triangle with sides 6, 8, 10 are similar with scale factor 2:  $\frac{6}{3} = \frac{8}{4} = \frac{10}{5} = 2$ .

## How do you use similar figures to find a missing length?

1. Match the corresponding sides.
2. Write a proportion:  $\frac{\text{new}}{\text{old}} = k$ .
3. Solve for the missing length.

**Worked example.** A 6-foot person casts a 4-foot shadow while a tree casts a 10-foot shadow. The sun makes similar triangles, so  $\frac{h}{6} = \frac{10}{4}$ . Then  $h = 6 \cdot 2.5 = 15$  feet.



Sun rays make similar right triangles.

To describe the sequence of moves between two similar figures, name the dilation and the rigid motions. For example, a triangle with vertices  $(1,1)$ ,  $(2,1)$ ,  $(1,3)$  maps to  $(-2,-2)$ ,  $(-4,-2)$ ,  $(-2,-6)$  by a dilation with scale factor 2 followed by a  $180^\circ$  rotation about the origin.

## Questions about dilations and similar figures

### What is a scale factor?

The scale factor is the number each length is multiplied by in a dilation. It equals the ratio of an image length to the matching original length.

### Is a dilated figure congruent to the original?

Only when the scale factor is 1. Otherwise the side lengths differ, so the figures are similar but not congruent.

### Do angles change in a dilation?

No. Dilations preserve angle measures and parallel lines. Only lengths (and areas) change.

### How does area change when the scale factor is $k$ ?

Area is multiplied by  $k^2$ . A scale factor of 3 makes the area 9 times as large.

## How do I know two shapes are similar?

Show equal corresponding angles and proportional corresponding sides, or describe a sequence of moves with a dilation that maps one onto the other.