

Grade 8 - Solving Linear Equations in One Variable - Lesson

Common Core 8.EE.C.7a, 8.EE.C.7b · maths-pdf.fr/en/

A linear equation in one variable is solved by doing the same operation to both sides until the variable is alone. Expand parentheses, combine like terms, collect the variable on one side, and divide. An equation can have one, none (a false statement like $5 = 9$), or infinitely many (a true statement like $7 = 7$) solutions.

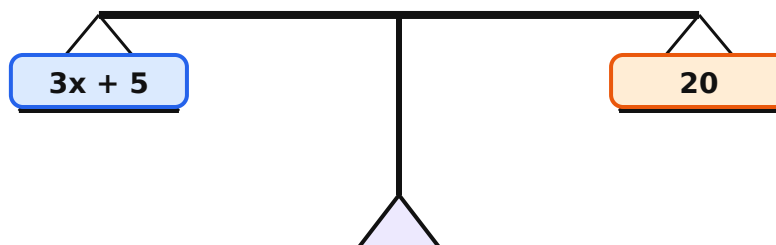
Grade 8 · Common Core State Standards: **8.EE.C.7a, 8.EE.C.7b**

Key facts

- Do the same thing to both sides to keep the equation balanced.
- Distribute first, then combine like terms, then collect variable terms.
- One solution: $x = \text{number}$. No solution: false statement. Infinitely many: true statement.
- Multiply by a common multiple to clear fractions and decimals.
- Always check by substituting your answer into the original equation.

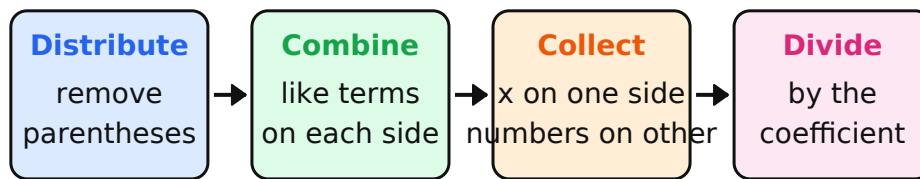
What is a linear equation in one variable?

A linear equation in one variable can be written as $ax+b=cx+d$. Solving it means finding the value of x that makes both sides equal. Think of a balance scale: you can add, subtract, multiply or divide both sides by the same number and the scale stays level.



Whatever you do to one side, do to the other.

How do you solve a multi-step equation?



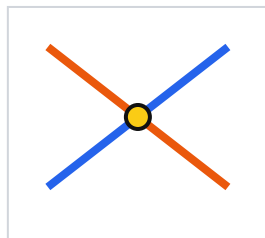
Four steps in a good order.

1. Use the distributive property to remove parentheses.
2. Combine like terms on each side.
3. Add or subtract to get the variable terms on one side and the numbers on the other.
4. Divide by the coefficient and check your answer.

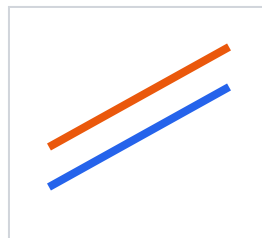
Worked example. Solve $4(x-3)=2x+6$. Distribute: $4x-12=2x+6$. Subtract $2x$: $2x-12=6$. Add 12: $2x=18$. Divide by 2: $x=9$. Check: $4(9-3)=24$ and $2\cdot 9+6=24$.

Number of solutions

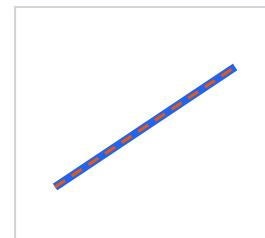
One solution: the variable terms differ, such as $3x+1=2x+6$. **No solution:** both sides simplify to a false statement like $5=9$. **Infinitely many solutions:** both sides simplify to a true statement like $7=7$.



One solution



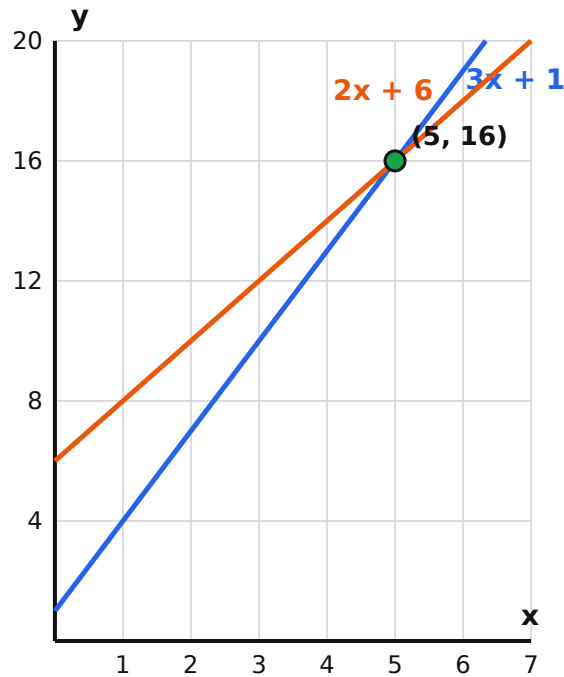
No solution



Infinite

Crossing once, parallel, or the same line.

What do the three outcomes look like?



The lines cross where $x = 5$.

Each side of an equation can be graphed as a line. One solution means the lines cross once. No solution means they are parallel. Infinitely many solutions means they are the same line. For example, $2(3x+1) = 6x+12$ simplifies to $6x+2 = 6x+12$; subtracting $6x$ gives $2 = 12$, which is false, so there is no solution.

With fractions or decimals, multiply every term by a common multiple to clear them. To solve $\frac{1}{2}x + 3 = \frac{1}{4}x + 5$, multiply by 4: $2x + 12 = x + 20$, so $x = 8$.

Questions about solving linear equations in one variable

How do I know if an equation has no solution?

Simplify both sides. If you reach a false statement like $2 = 12$ with no variable left, there is no solution.

What does infinitely many solutions mean?

Every value of x works. After simplifying you get a true statement like $7 = 7$, for example in $3(x+4) = 3x+12$.

Which side should the variable end up on?

Either side works. Moving the smaller variable term avoids negative coefficients, but the answer is the same.

How do I get rid of fractions in an equation?

Multiply every term on both sides by the least common denominator. For $\frac{x}{2} + 3 = \frac{x}{4} + 5$, multiply by 4.

How can I check my answer?

Substitute the solution for x in the original equation and compute both sides. If they are equal, the solution is correct.