

Grade 8 - Proportional Relationships and Slope - Lesson

Common Core 8.EE.B.5 · maths-pdf.fr/en/

A proportional relationship has a constant ratio between two quantities, so it can be written $y = kx$. Its graph is a straight line through the origin. The constant k is the unit rate, and it equals the slope of the line, which is rise divided by run between any two points.

Grade 8 · Common Core State Standards: **8.EE.B.5**

Key facts

- Equation: $y = kx$, where $k = \frac{y}{x}$ is the unit rate.
- The graph is a straight line through the origin $(0,0)$.
- Slope = $\frac{\text{rise}}{\text{run}}$, the same between any two points on the line.
- A greater unit rate means a steeper line.
- In a table, check that $\frac{y}{x}$ is constant.

What is a proportional relationship?

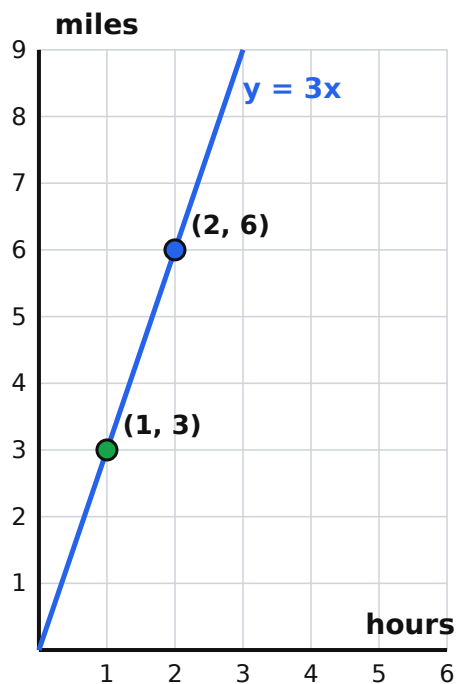
Two quantities are in a proportional relationship when one is always a constant multiple of the other. That constant is the **unit rate**, written k in the equation $y = kx$.

Proportional relationship

A relationship is **proportional** if $\frac{y}{x}$ is the same for every pair of values. Its equation is $y = kx$, its graph is a straight line through the origin $(0,0)$, and the constant k is the unit rate and the **slope** of the line.

How does the graph show the unit rate?

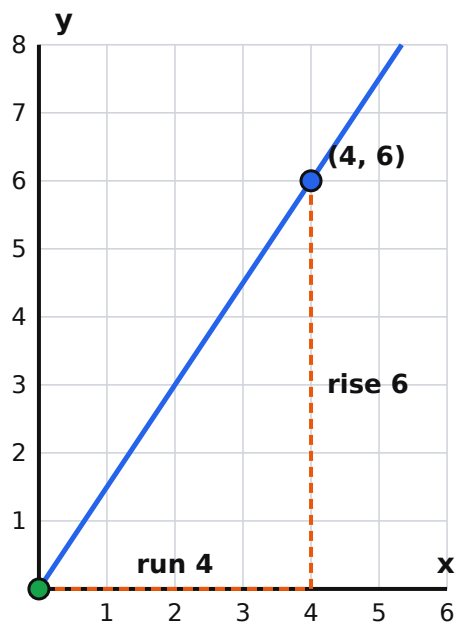
Pick any point on the line and read its coordinates. For the line below, the point $(2,6)$ shows 6 miles in 2 hours. The unit rate is the value at $x=1$, which is 3 miles per hour.



Walking at 3 miles per hour.

What is slope?

Slope measures how steep a line is. Move from one point to another: the vertical change is the **rise** and the horizontal change is the **run**. The slope is $\frac{\text{rise}}{\text{run}}$, and for a proportional relationship it equals the unit rate.



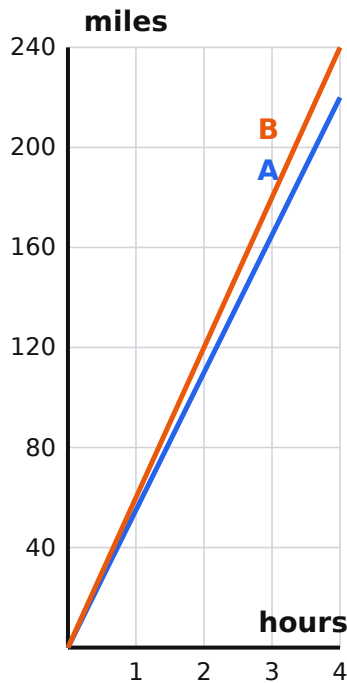
Rise 6 over run 4 gives slope $\frac{3}{2}$.

Here the slope is $\frac{6}{4} = \frac{3}{2}$, so $y = \frac{3}{2}x$. A steeper line has a greater slope. A line that goes down

from left to right has a negative slope.

How do you compare two proportional relationships?

Find the unit rate of each, then compare. Suppose Car A is shown by $y = 55x$ and Car B travels 120 miles in 2 hours. Car B's rate is $120 \div 2 = 60$ miles per hour, which is faster than 55.



Car B's line is steeper than car A's line.

1. Find the unit rate of each relationship (divide y by x).
2. Check that each graph passes through the origin.
3. Compare the unit rates: the greater rate is the steeper line.

x	y
1	5
2	9
3	13

Not proportional: the table $x = 1, 2, 3$ with $y = 5, 9, 13$ has $\frac{y}{x}$ values 5, 4.5 and 4.33. They are not equal, and the line would not pass through the origin.

Questions about proportional relationships and slope

How can I tell if a table is proportional?

Divide each y by its x . If every quotient is the same number, the relationship is proportional. If they differ, it is not.

Why does a proportional graph go through the origin?

When $x=0$, $y=k \cdot 0=0$. So the point $(0,0)$ is always on the line.

Is slope the same as unit rate?

For a proportional relationship, yes. The unit rate is the y -value when $x=1$, which is also rise over run. In other lines, slope is a rate of change.

Can the slope be a fraction?

Yes. A line through $(0,0)$ and $(4,6)$ has slope $\frac{6}{4}=\frac{3}{2}$. The unit rate is 1.5.

How do I compare a graph and an equation?

Read the unit rate from the graph at $x=1$ or by rise over run, and read k from the equation. Then compare the two numbers.