

Grade 8 - The Pythagorean Theorem - Lesson

Common Core 8.G.B.6, 8.G.B.7 · maths-pdf.fr/en/

The Pythagorean theorem says that in a right triangle with legs a and b and hypotenuse c , $a^2 + b^2 = c^2$. Use it to find a missing side, to test whether a triangle is a right triangle, and to solve real problems involving diagonals, ladders and ramps.

Grade 8 · Common Core State Standards: **8.G.B.6**, **8.G.B.7**

Key facts

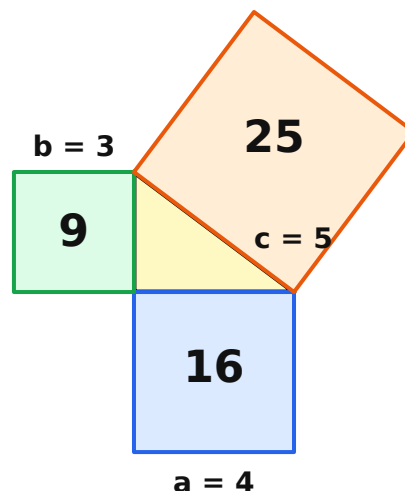
- $a^2 + b^2 = c^2$, where c is the hypotenuse.
- To find a leg: $a^2 = c^2 - b^2$.
- Converse: if $a^2 + b^2 = c^2$, the triangle has a right angle.
- Common triples: 3-4-5, 5-12-13, 8-15-17 and their multiples.
- The theorem works in 3D: the space diagonal satisfies $d^2 = a^2 + b^2 + c^2$.

What is the Pythagorean theorem?

In a right triangle, the side opposite the right angle is the **hypotenuse**, and it is the longest side. The other two sides are the **legs**. The Pythagorean theorem links all three sides.

Pythagorean theorem

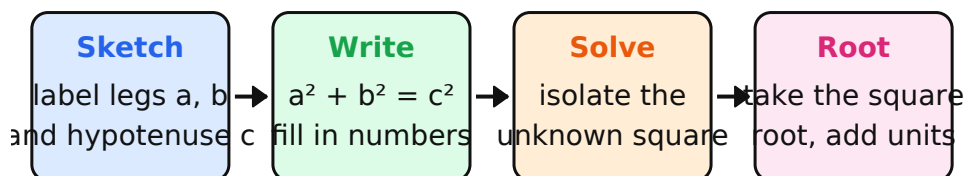
If a triangle has legs a and b and hypotenuse c , then $a^2 + b^2 = c^2$. The **converse** is also true: if $a^2 + b^2 = c^2$, the triangle is a right triangle.



3-4-5: the squares on the legs add up to the square on the hypotenuse.

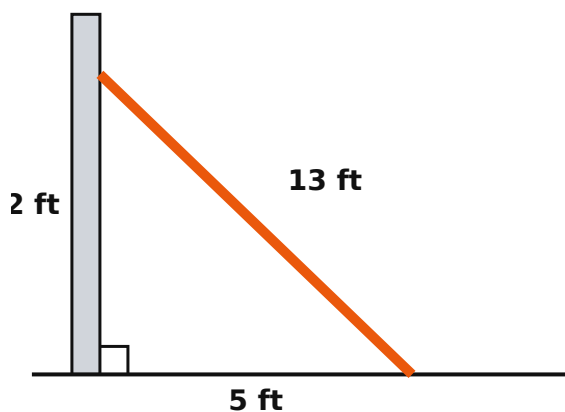
The picture shows why it works for the 3-4-5 triangle: the squares on the legs have areas $3^2=9$ and $4^2=16$, and together they equal the square on the hypotenuse, $5^2=25$.

How do you find a missing side?



Four steps.

Worked example. A 13-foot ladder leans on a wall with its base 5 feet from the wall. How high does it reach? The ladder is the hypotenuse, so $5^2 + h^2 = 13^2$. Then $25 + h^2 = 169$, so $h^2 = 144$ and $h = 12$ feet.

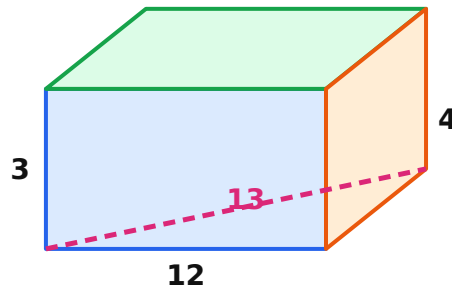


The ladder reaches 12 ft up the wall.

How do you use the theorem in real problems?

Look for a right angle hidden in the situation: a diagonal of a rectangle, a ladder, a ramp, a diagonal path, or the distance between two points. If the answer is not a whole number, estimate it. For example, the legs 7 and 9 give $c^2 = 49 + 81 = 130$, so $c = \sqrt{130} \approx 11.4$.

The theorem also works in three dimensions. The space diagonal of a box with edges 3, 4 and 12 satisfies $d^2 = 3^2 + 4^2 + 12^2 = 169$, so $d = 13$. First find the diagonal of the floor, then use it as one leg of a second right triangle whose other leg is the height of the box.



Space diagonal of a 12 by 4 by 3 box.

Whenever you meet a new problem, ask three questions. Is there a right angle? Which side is the hypotenuse? Is the answer reasonable, meaning longer than each leg but shorter than the sum of the two legs? For example, the hypotenuse of a triangle with legs 6 and 8 must be between 8 and 14, and it is 10.

Testing a triangle

To check whether sides $a < b < c$ form a right triangle, compare $a^2 + b^2$ with c^2 . For 20, 21, 29: $400 + 441 = 841 = 29^2$, so it is a right triangle.

Questions about the pythagorean theorem

Which side is the hypotenuse?

It is the side opposite the right angle and always the longest side. In $a^2 + b^2 = c^2$, c is the hypotenuse.

Can I use the theorem on any triangle?

No. It only works for right triangles. For other triangles, the sum of the squares of two sides does not equal the square of the third.

What is a Pythagorean triple?

A set of three whole numbers that satisfy $a^2 + b^2 = c^2$, such as 3-4-5, 5-12-13 and 8-15-17.

What if the square root is not a whole number?

Leave it as a square root or estimate it. For $c^2 = 130$, $c \approx 11.4$ because $11.4^2 = 129.96$.

How do I use the converse?

Square all three sides. If the two smaller squares add up to the largest square, the triangle is a right triangle.