

Grade 8 - Rational and Irrational Numbers - Lesson

Common Core 8.NS.A.1, 8.NS.A.2 · maths-pdf.fr/en/

A rational number is any number that can be written as a fraction of two integers; its decimal form stops or repeats. An irrational number, such as the square root of 2 or pi, has a decimal that never stops and never repeats. Together they make up the real numbers.

Grade 8 · Common Core State Standards: **8.NS.A.1, 8.NS.A.2**

Key facts

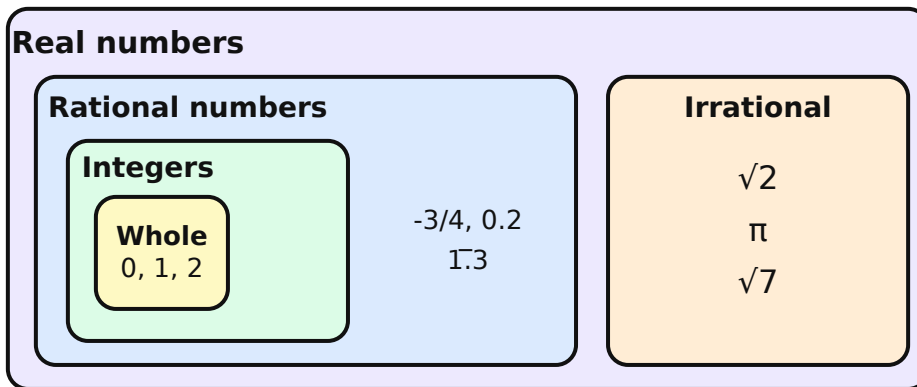
- Terminating and repeating decimals are rational; every rational number can be written as a fraction.
- Square roots of non-perfect squares, and π , are irrational.
- To write $0.\overline{7}$ as a fraction, multiply by 10, subtract, and divide: $\frac{7}{9}$.
- Estimate $\sqrt{n}$ by finding the two perfect squares around n , then refine to tenths and hundredths.
- Compare real numbers by converting them to decimals with the same number of places.

What is a rational number?

A rational number is any number you can write as a fraction of two integers. Fractions, integers, terminating decimals and repeating decimals are all rational.

Rational and irrational

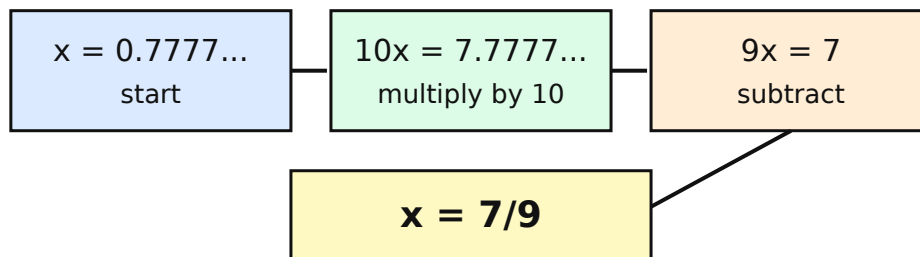
A **rational number** can be written as $\frac{a}{b}$ with integers a and b , $b \neq 0$. Its decimal form stops or repeats. An **irrational number** cannot be written that way; its decimal never stops and never repeats.



Every real number is either rational or irrational, never both.

How do you turn a repeating decimal into a fraction?

Let x equal the decimal, multiply by a power of 10 so the repeating blocks line up, and subtract. The repeating tail cancels.



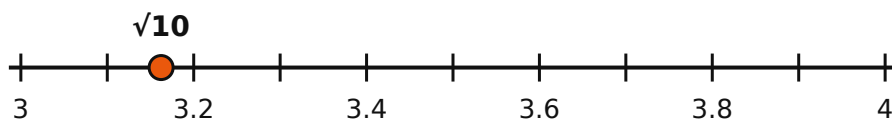
Multiply by 10 so the repeating tails line up, then subtract.

1. Set $x = 0.\bar{7}$.
2. Multiply by 10: $10x = 7.\bar{7}$.
3. Subtract: $10x - x = 7.\bar{7} - 0.\bar{7}$, so $9x = 7$.
4. Divide: $x = \frac{7}{9}$.

If two digits repeat, multiply by 100 instead. For example, $0.\overline{45} = \frac{45}{99} = \frac{5}{11}$.

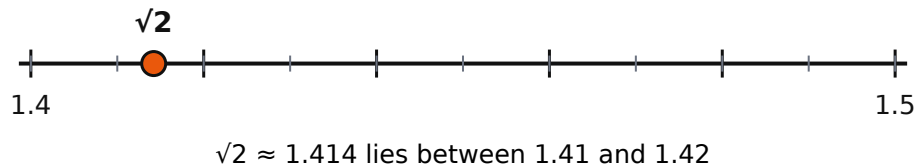
How do you estimate an irrational number?

Square roots of numbers that are not perfect squares are irrational. To place $\sqrt{10}$, find the perfect squares on each side: $3^2 = 9$ and $4^2 = 16$, so $3 < \sqrt{10} < 4$. Since 10 is only 1 more than 9, the root is close to 3. Testing $3.1^2 = 9.61$ and $3.2^2 = 10.24$ shows $3.1 < \sqrt{10} < 3.2$.



$\sqrt{10}$ is between 3 and 4, close to 3.2

Keep zooming in and the same idea gives more digits: $\sqrt{2} = 1.41421356\dots$ never ends and never repeats.



Which numbers are irrational?

Number	Type	Reason
$\sqrt{49}$	Rational	$\sqrt{49} = 7$
$\sqrt{7}$	Irrational	7 is not a perfect square
π	Irrational	Decimal never repeats
$0.\overline{36}$	Rational	$= \frac{4}{11}$

How do you compare and order real numbers?

Write each number as a decimal to the same number of places, then order the decimals. To order $\sqrt{7}$, 2.6 and $\frac{8}{3}$, find $\sqrt{7} \approx 2.646$ and $\frac{8}{3} = 2.666\dots$. Now the order is clear: $2.6 < \sqrt{7} < \frac{8}{3}$.

Remember two safe rules. First, a sum or product of two rational numbers is always rational. Second, a nonzero rational number times an irrational number, such as 3π , is irrational.

Questions about rational and irrational numbers

Is zero a rational number?

Yes. Zero equals $\frac{0}{1}$, a fraction of two integers, so it is rational. It is also a whole number and an integer.

Is a repeating decimal always rational?

Yes. Any decimal with a block of digits that repeats forever can be turned into a fraction by multiplying by a power of 10 and subtracting. For example, $0.\overline{3} = \frac{1}{3}$.

Why is the square root of 2 irrational?

No fraction squared equals exactly 2, and its decimal form 1.41421356... never repeats.

Mathematicians prove this with a contradiction argument that you will see in high school.

Is $\frac{22}{7}$ equal to pi?

No. $\frac{22}{7} = 3.142857\ldots$ repeats, while $\pi = 3.14159\ldots$ never repeats. The fraction is a handy rational approximation, slightly larger than π .

How do I compare a square root and a decimal?

Square both numbers, or estimate the root to two decimal places. For instance $\sqrt{30} \approx 5.48$, which is less than 5.5, because $5.5^2 = 30.25 > 30$.