

Grade 8 - Slope and Similar Triangles - Lesson

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The slope of a line is the same between any two points because the right triangles formed by rise and run are similar. A line with slope m through the origin has equation $y = mx$, and a line with slope m and y-intercept b has equation $y = mx + b$. Slope equals rise divided by run.

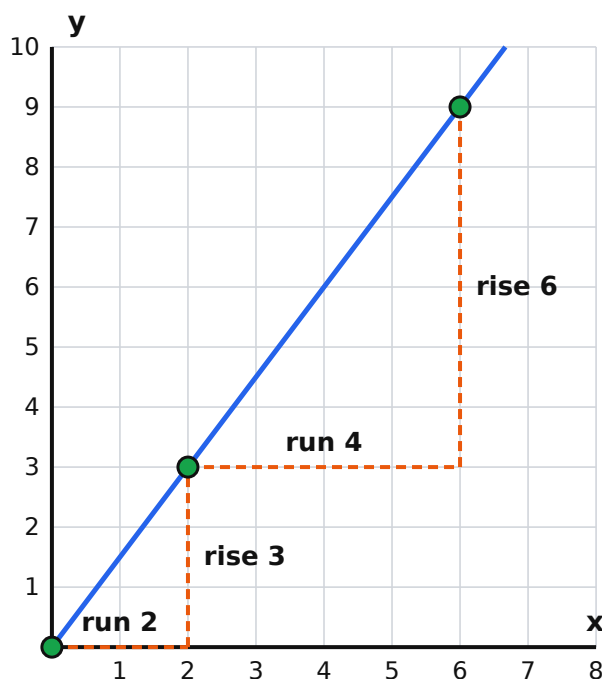
Grade 8 · Common Core State Standards: **8.EE.B.6**

Key facts

- Slope $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.
- Any two points on a line give the same slope because the triangles are similar.
- $y = mx$ goes through the origin; $y = mx + b$ crosses the y-axis at $(0, b)$.
- Parallel lines have equal slopes; horizontal lines have slope 0.
- To write an equation, find m , then solve for b with a point.

Why is the slope the same between any two points on a line?

Pick two points on a line and draw a right triangle with a horizontal leg (the run) and a vertical leg (the rise). Any two such triangles on the same line have the same angles, so they are similar. Similar triangles have proportional sides, so $\frac{\text{rise}}{\text{run}}$ is the same for both.



The small triangle has rise 3 and run 2; the large one has rise 6 and run 4.

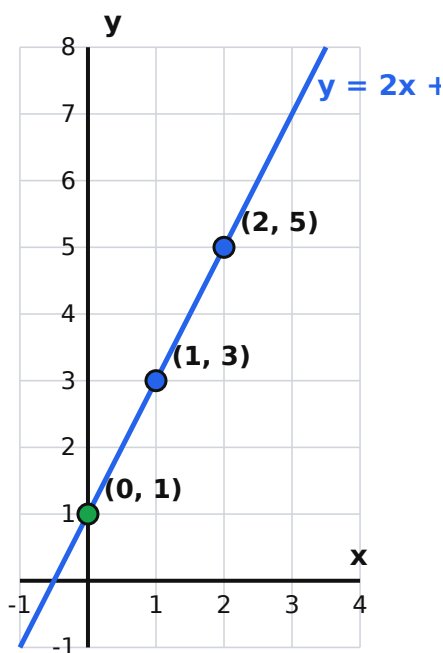
Small triangle: $\frac{3}{2}$. Large triangle: $\frac{6}{4} = \frac{3}{2}$. The slope is constant, so we can call it m .

Equations of a line

A line through the origin with slope m has equation $y = mx$. A line with slope m that crosses the y-axis at $(0, b)$ has equation $y = mx + b$. The number b is the **y-intercept**.

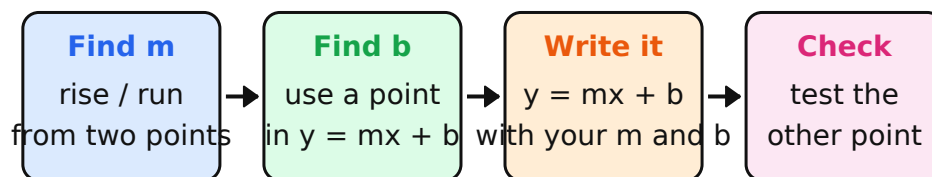
How do you read slope and intercept from an equation?

In $y = 2x + 1$, the slope is $m = 2$ and the y-intercept is $b = 1$. Start at $(0, 1)$, go up 2 and right 1 to reach $(1, 3)$, then repeat.



Intercept 1, slope 2.

How do you write the equation of a line?

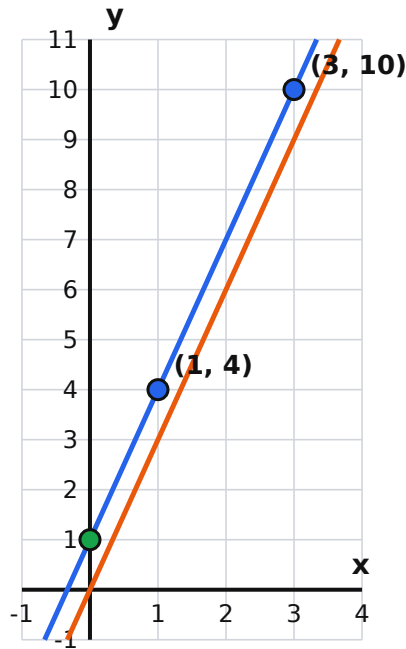


A four step method.

1. Find the slope from two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Substitute one point and m into $y = mx + b$ to find b .

3. Write the equation.
4. Check with the other point.

Worked example. A line passes through (1,4) and (3,10). The slope is $\frac{10-4}{3-1} = 3$. Substitute (1,4): $4 = 3 \cdot 1 + b$, so $b = 1$. The equation is $y = 3x + 1$. Check: $3 \cdot 3 + 1 = 10$.



The blue line $y = 3x + 1$ is the orange line $y = 3x$ moved up 1.

Lines with the same slope are parallel. A horizontal line has slope 0, so its equation is $y = b$. Negative slopes go down from left to right.

Questions about slope and similar triangles

What are rise and run?

Rise is the vertical change between two points, and run is the horizontal change. Slope is rise divided by run.

Does it matter which point I call the first point?

No, as long as you subtract in the same order for y and x . $\frac{10-4}{3-1}$ and $\frac{4-10}{1-3}$ both equal 3.

What is the slope of a horizontal line?

Zero, because the rise is 0. A horizontal line through (0,5) is $y = 5$. A vertical line has undefined slope.

How can I find b if I know the slope and a point?

Substitute the point into $y = mx + b$ and solve. For slope 2 and point (3,11): $11 = 6 + b$, so $b = 5$.

How do I rewrite $2x + y = 6$ as $y = mx + b$?

Subtract $2x$ from both sides: $y = -2x + 6$. Now the slope is -2 and the y-intercept is 6.