

Precalculus - Radians and the Unit Circle - Lesson

Common Core F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4 · maths-pdf.fr/en/

A radian is the angle that cuts off an arc equal to the radius, so 180 degrees equals π radians. On the unit circle, an angle θ in standard position meets the circle at $(\cos \theta, \sin \theta)$. Use reference angles and quadrant signs to find exact trig values, and $s = r \theta$ for arc length.

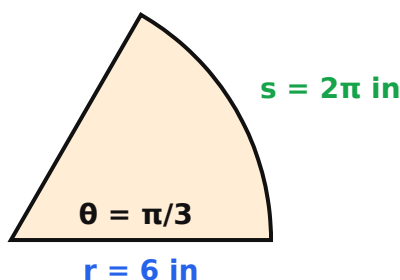
Precalculus · Common Core State Standards: **F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4**

Key facts

- π radians = 180 degrees; convert with the factor $\pi/180$ or $180/\pi$.
- Arc length $s = r \theta$ and sector area $A = (1/2) r^2 \theta$, with θ in radians.
- The point on the unit circle at angle θ is $(\cos \theta, \sin \theta)$.
- Reference angle: the acute angle to the x-axis; the quadrant sets the sign.
- Coterminal angles differ by 2π and have the same trig values.

What is a radian?

One **radian** is the central angle that cuts off an arc equal in length to the radius. Because a full circle has circumference $2\pi r$, a full turn is 2π radians, so π radians equals 180° . To convert, multiply degrees by $\frac{\pi}{180}$ or radians by $\frac{180}{\pi}$. For example $60^\circ = \frac{\pi}{3}$ and $\frac{5\pi}{4} = 225^\circ$.



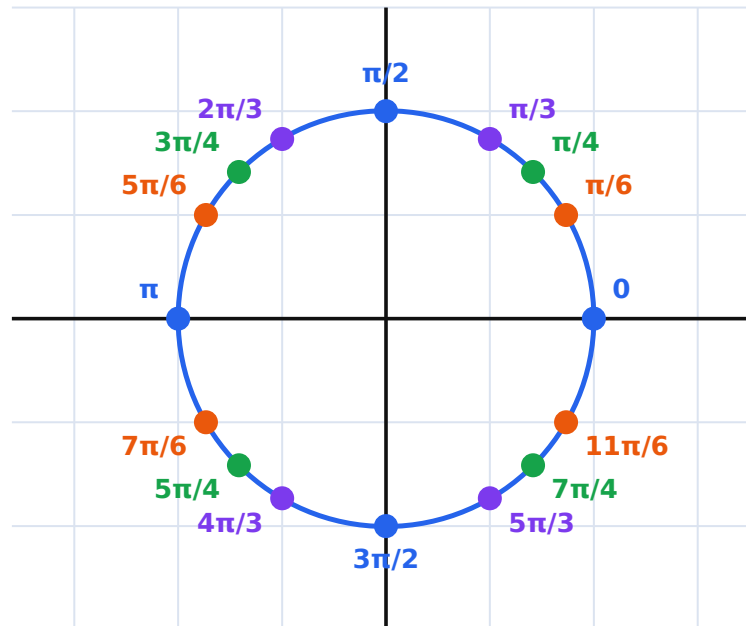
On a circle of radius 6 inches, an angle of $\frac{\pi}{3}$ cuts off an arc of $s = r\theta = 2\pi \approx 6.28$ inches.

Arc length and sector area

With θ in radians: arc length $s = r\theta$ and sector area $A = \frac{1}{2}r^2\theta$. These formulas fail if the angle is in degrees.

What is the unit circle?

The unit circle has radius 1 and center at the origin. For an angle θ in standard position (starting on the positive x -axis and turning counterclockwise), the terminal side meets the circle at the point $(\cos\theta, \sin\theta)$. Therefore $\tan\theta = \frac{\sin\theta}{\cos\theta}$, the slope of the terminal side.



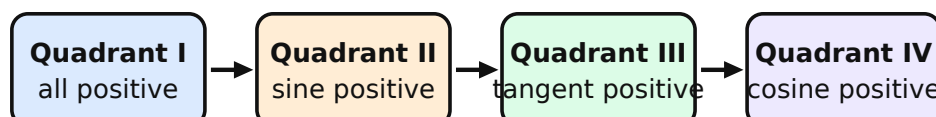
The special angles and their positions on the unit circle.

Angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

How do you find trig values for any angle?

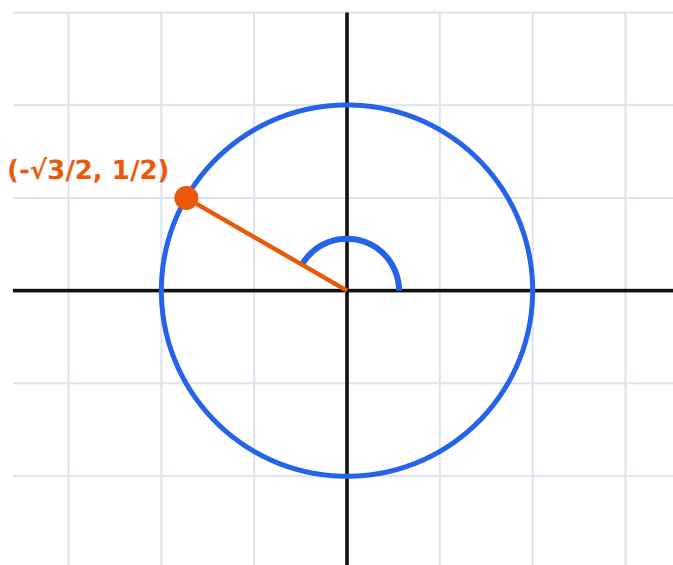
1. Find the **reference angle**: the acute angle between the terminal side and the x -axis.
2. Use the special value for that reference angle.
3. Choose the sign by quadrant: sine is positive in quadrants I and II, cosine in I and IV, tangent in I and III.

Example: $\sin\frac{5\pi}{6}$. The terminal side is in quadrant II with reference angle $\frac{\pi}{6}$, so $\sin\frac{5\pi}{6} = \frac{1}{2}$ and $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$.



Which function is positive in each quadrant.

A quick memory aid is the phrase All Students Take Calculus, listing the positive functions in quadrants I through IV. Think of an angle such as 210° : it lies in quadrant III, its reference angle is 30° , and because only tangent is positive there, $\sin 210^\circ = -\frac{1}{2}$ and $\cos 210^\circ = -\frac{\sqrt{3}}{2}$. This three-step method (quadrant, reference angle, sign) works for every angle, even for angles larger than a full turn or negative angles, once you subtract or add multiples of 2π first.



At $\theta = \frac{5\pi}{6}$ the point is $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$.

Angles that differ by a full turn are **coterminal** and share the same trig values: $\sin(\theta + 2\pi k) = \sin \theta$. Sine is odd, $\sin(-\theta) = -\sin \theta$, and cosine is even, $\cos(-\theta) = \cos \theta$. Combined with the Pythagorean relation $\sin^2 \theta + \cos^2 \theta = 1$, one known value lets you find the others.

Questions about radians and the unit circle

Why use radians instead of degrees?

Radians make formulas simple: arc length is $s = r\theta$ and sector area is $(1/2)r^2\theta$. They also make calculus formulas for sine and cosine clean.

How do you convert degrees to radians?

Multiply by $\pi/180$. For 150 degrees, 150 times $\pi/180$ equals $5\pi/6$.

What is a reference angle?

It is the acute angle between the terminal side of the angle and the x-axis. It tells you the size

of the trig values, and the quadrant tells you the sign.

How do I remember which trig functions are positive?

Use All, Sine, Tangent, Cosine by quadrant: all positive in I, only sine in II, only tangent in III, only cosine in IV.

What does coterminal mean?

Coterminal angles have the same terminal side. They differ by whole multiples of 2π radians (360 degrees), so their sine and cosine are equal.