

Precalculus - Radians and the Unit Circle - Answer Key

Common Core F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4 · maths-pdf.fr/en/

Use this key to check each angle or trig value. Solutions show the conversion factor, the reference angle and the quadrant sign, with final answers in bold.

Precalculus · Common Core State Standards: **F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4**

Exercise 1 - Degrees to radians

$$45 \cdot \frac{\pi}{180} = \frac{\pi}{4} \cdot 150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$$

$$\frac{\pi}{4} \text{ and } \frac{5\pi}{6}$$

Exercise 2 - Radians to degrees

Multiply by $\frac{180}{\pi}$: 30° and 225° .

$$30^\circ \text{ and } 225^\circ$$

Exercise 3 - Calculator conversion

$$3 \cdot \frac{180}{\pi} \approx 171.89^\circ \cdot 100 \cdot \frac{\pi}{180} \approx 1.75$$

$$171.89^\circ \text{ and } 1.75 \text{ rad}$$

Exercise 4 - Coterminal angles

$$50 + 360 = 410^\circ; 50 - 360 = -310^\circ. \text{ For } \frac{7\pi}{4}: \frac{7\pi}{4} - 2\pi = -\frac{\pi}{4}; \frac{7\pi}{4} + 2\pi = \frac{15\pi}{4}$$

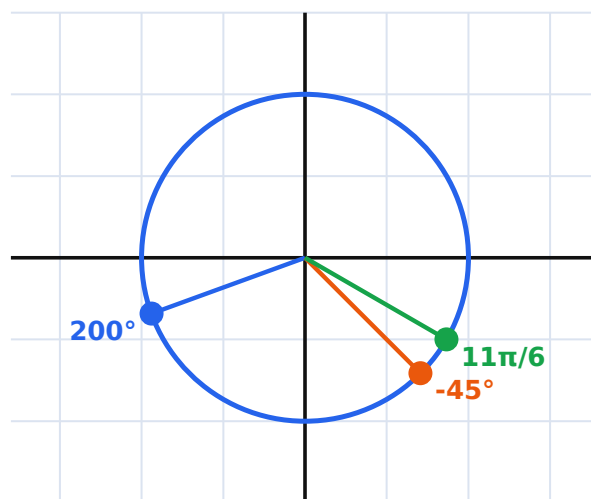
$$410^\circ, -310^\circ; \frac{15\pi}{4}, -\frac{\pi}{4}$$

Exercise 5 - Reference angle

$$180^\circ - 150^\circ = 30^\circ \cdot \frac{5\pi}{4} - \pi = \frac{\pi}{4}$$

30° and $\frac{\pi}{4}$

Exercise 6 - Quadrants



200 degrees is in quadrant III; the other two are in quadrant IV.

200° is between 180° and 270° . -45° turns clockwise into the lower right. $\frac{11\pi}{6} = 330^\circ$.

III, IV, IV

Exercise 7 - Arc length

$$s = r\theta = 6 \cdot \frac{\pi}{3} = 2\pi \approx 6.28.$$

$$2\pi \approx 6.28 \text{ in}$$

Exercise 8 - Sector area

$$A = \frac{1}{2}(10)^2 \cdot \frac{2\pi}{5} = 20\pi \approx 62.83.$$

$$20\pi \approx 62.83 \text{ ft}^2$$

Exercise 9 - Find the angle

$$\theta = \frac{s}{r} = \frac{15}{6} = 2.5 \text{ rad. In degrees: } 2.5 \cdot \frac{180}{\pi} \approx 143.2^\circ.$$

$$2.5 \text{ rad} \approx 143.2^\circ$$

Exercise 10 - Exact values, first quadrant

$$\sin 30^\circ = \frac{1}{2}, \cos 45^\circ = \frac{\sqrt{2}}{2}, \tan 60^\circ = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$

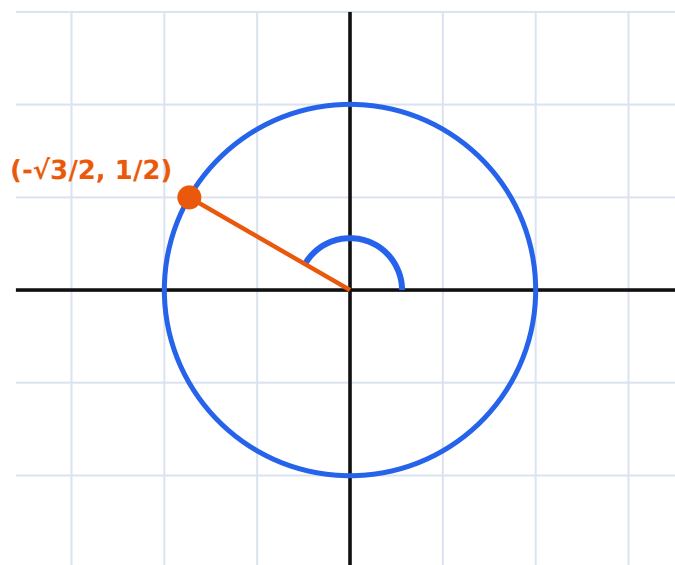
$$\frac{1}{2}, \frac{\sqrt{2}}{2}, \sqrt{3}$$

Exercise 11 - Exact values, other quadrants

$\frac{2\pi}{3}$: reference $\frac{\pi}{3}$, quadrant II, cosine negative: $-\frac{1}{2}$. $\frac{5\pi}{4}$: reference $\frac{\pi}{4}$, quadrant III: $-\frac{\sqrt{2}}{2}$. $\frac{3\pi}{4}$: quadrant II: $\tan = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$.

$$-\frac{1}{2}, -\frac{\sqrt{2}}{2}, -1$$

Exercise 12 - Point on the unit circle



The point is $(-\sqrt{3}/2, 1/2)$.

The reference angle is $\frac{\pi}{6}$ in quadrant II, so $\cos \theta = -\frac{\sqrt{3}}{2}$ and $\sin \theta = \frac{1}{2}$. $\tan \theta = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$.

$$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right); \tan \theta = -\frac{\sqrt{3}}{3}$$

Exercise 13 - Find the other values

$$\cos^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}. \text{ In quadrant II cosine is negative: } -\frac{4}{5}. \tan \theta = \frac{3/5}{-4/5}.$$

$$\cos \theta = -\frac{4}{5}, \tan \theta = -\frac{3}{4}$$

Exercise 14 - Another quadrant

$$\sin^2 \theta = 1 - \frac{25}{169} = \frac{144}{169}. \text{ Quadrant III: sine is negative, } -\frac{12}{13}. \tan \theta = \frac{-12/13}{-5/13}.$$

$$\sin \theta = -\frac{12}{13}, \tan \theta = \frac{12}{5}$$

Exercise 15 - Negative angles

$$\text{Sine is odd: } -\sin \frac{\pi}{6} = -\frac{1}{2}. \text{ Cosine is even: } \cos \frac{\pi}{3} = \frac{1}{2}.$$

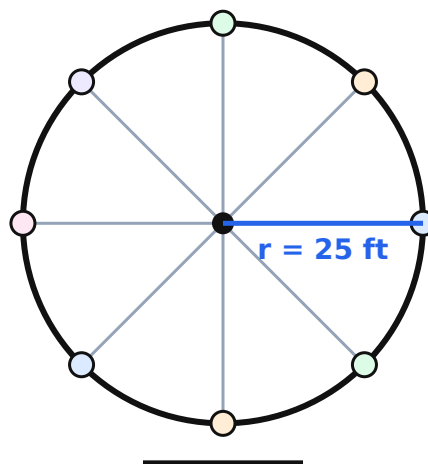
$$-\frac{1}{2} \text{ and } \frac{1}{2}$$

Exercise 16 - Large and negative angles

$$\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}, \text{ so } \sin = \frac{\sqrt{3}}{2}. \cos\left(-\frac{5\pi}{6}\right) = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}.$$

$$\frac{\sqrt{3}}{2} \text{ and } -\frac{\sqrt{3}}{2}$$

Exercise 17 - Ferris wheel

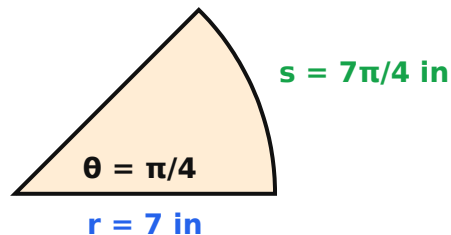


Speed $\pi/10$ radians per second.

$$3 \cdot 2\pi = 6\pi \text{ rad per minute, so } \frac{6\pi}{60} = \frac{\pi}{10} \text{ rad/s. In 10 s: } \theta = \pi, \text{ so } s = 25\pi \approx 78.5.$$

$$\frac{\pi}{10} \text{ rad/s}; 25\pi \approx 78.5 \text{ ft}$$

Exercise 18 - Pizza slice



One slice.

$$\theta = \frac{2\pi}{8} = \frac{\pi}{4}. \text{ Crust: } s = 7 \cdot \frac{\pi}{4} \approx 5.5. \text{ Area: } \frac{1}{2}(49)\frac{\pi}{4} = \frac{49\pi}{8} \approx 19.24.$$

$$5.5 \text{ in}; 19.24 \text{ in}^2$$

Exercise 19 - Windshield wiper

$$120^\circ = \frac{2\pi}{3}. A = \frac{1}{2}(18)^2 \cdot \frac{2\pi}{3} = 108\pi \approx 339.3.$$

$$108\pi \approx 339.3 \text{ in}^2$$

Exercise 20 - Distance on Earth

$$5^\circ = \frac{5\pi}{180} = \frac{\pi}{36} \text{ rad. } s = 3959 \cdot \frac{\pi}{36} \approx 345.$$

About 345 miles