

Precalculus - The Binomial Theorem - Answer Key

Common Core A-APR.C.5 · maths-pdf.fr/en/

Use this key to check each expansion and coefficient. Solutions show the row of Pascal's triangle or the general term used, with final answers in bold.

Precalculus · Common Core State Standards: **A-APR.C.5**

Exercise 1 - Factorials

$6! = 720$. $0! = 1$ by definition. $\frac{8!}{6!} = 8 \cdot 7 = 56$.

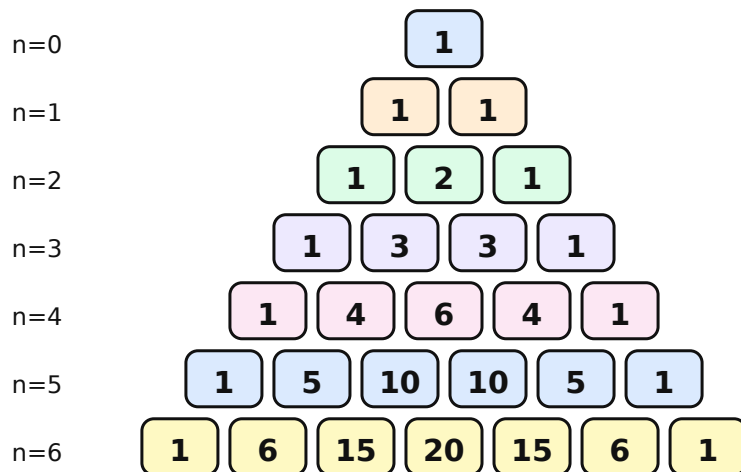
720, 1, 56

Exercise 2 - Binomial coefficients

$C(7,3) = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$. $C(10,2) = \frac{10 \cdot 9}{2} = 45$. $C(9,9) = 1$. $C(12,10) = C(12,2) = \frac{12 \cdot 11}{2} = 66$.

35, 45, 1, 66

Exercise 3 - Pascal's triangle



Row 6 is shaded.

Add neighbors in row 5 (1,5,10,10,5,1): 1, 1+5, 5+10, 10+10, 10+5, 5+1, 1.

1, 6, 15, 20, 15, 6, 1

Exercise 4 - Expand $(x + y)^4$

Row 4 of the triangle is 1,4,6,4,1.

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Exercise 5 - Expand $(x + 2)^5$

k	0	1	2	3	4	5
C(5,k)	1	5	10	10	5	1
$x^{(5-k)}$	x^5	x^4	x^3	x^2	x	1
2^k	1	2	4	8	16	32
term	$1x^5$	$10x^4$	$40x^3$	$80x^2$	$80x$	32

C(5,k), powers of x and powers of 2.

Multiply the row-5 coefficients by powers of 2 and add the powers of x.

$$x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

Exercise 6 - Expand $(3x + 1)^4$

$$(3x)^4 + 4(3x)^3(1) + 6(3x)^2(1)^2 + 4(3x)(1)^3 + 1^4.$$

$$81x^4 + 108x^3 + 54x^2 + 12x + 1$$

Exercise 7 - Expand $(x - 3)^3$

Row 3 is 1,3,3,1. $x^3 + 3x^2(-3) + 3x(9) + (-27)$.

$$x^3 - 9x^2 + 27x - 27$$

Exercise 8 - Find a single term

The 4th term has $k=3$: $C(8,3) x^5 \cdot 3^3 = 56 \cdot 27 x^5$.

$$1512x^5$$

Exercise 9 - Find a coefficient

Power x^3 needs $6-k=3$, so $k=3$: $C(6,3)(2x)^3(1)^3=20\cdot 8x^3$.

160

Exercise 10 - Coefficient with a negative

x^4 means $7-k=4$, $k=3$. $C(7,3)(-2)^3=35(-8)$.

-280

Exercise 11 - Constant term

The general term is $C(6,k)x^{6-k}\left(\frac{2}{x}\right)^k=C(6,k)2^kx^{6-2k}$. The exponent is 0 when $k=3$:
 $C(6,3)\cdot 8=20\cdot 8$.

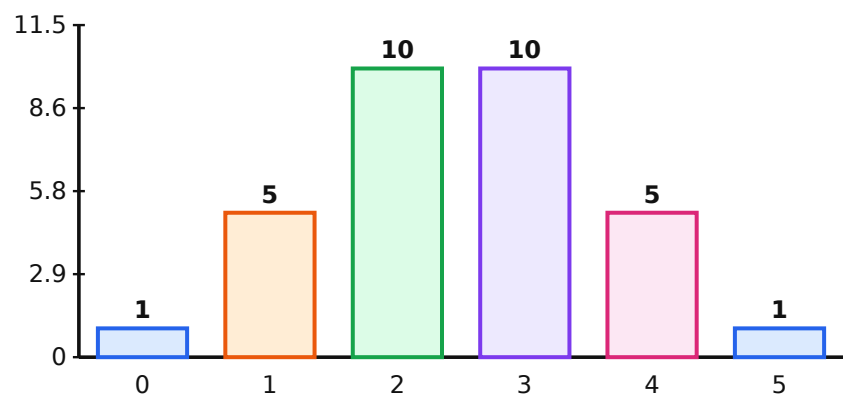
160

Exercise 12 - Sum of coefficients

Set $x=1$: $(3-1)^5=2^5$.

32

Exercise 13 - Row sums



The bars are 1, 5, 10, 10, 5, 1.

$1+5+10+10+5+1=32=2^5$. This matches $(1+1)^5$.

32

Exercise 14 - Middle term

There are 11 terms; the middle is $k=5$: $C(10,5)x^5y^5$.

$$252x^5y^5$$

Exercise 15 - Approximate a power

$$1+5(0.02)+10(0.02)^2+10(0.02)^3=1+0.1+0.004+0.00008=1.10408.$$

1.10408, **very close to** 1.10408

Exercise 16 - Complex number

$$1+4i+6i^2+4i^3+i^4=1+4i-6-4i+1.$$

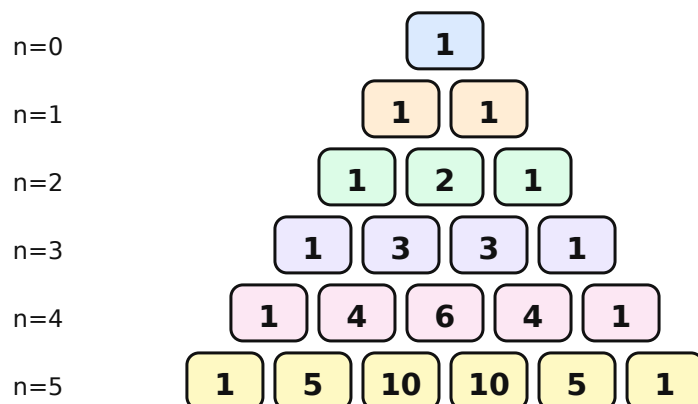
$$-4$$

Exercise 17 - Choose a committee

Order does not matter: $C(8,3)=\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$.

56 committees

Exercise 18 - Coin tosses

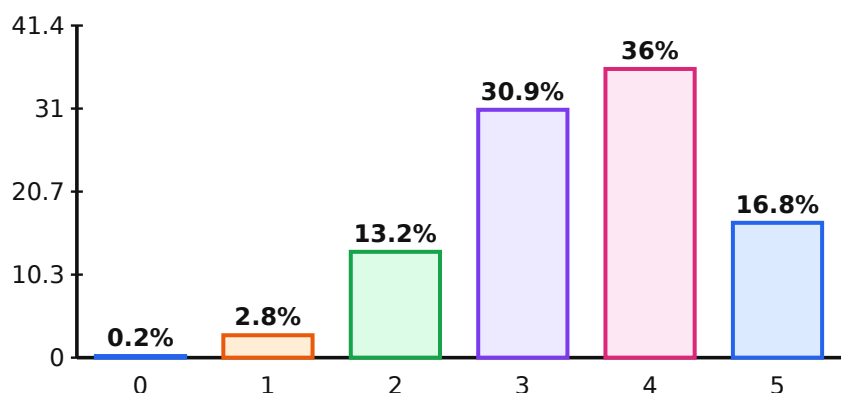


10 of the 32 outcomes have 3 heads.

There are $2^5 = 32$ equally likely outcomes, and $C(5,3) = 10$ have exactly 3 heads.

$$\frac{10}{32} = \frac{5}{16}$$

Exercise 19 - Free throws



The bar at 4 is about 36 percent.

The term of $(0.7+0.3)^5$ with 4 makes is $C(5,4)(0.7)^4(0.3)^1 = 5(0.2401)(0.3) = 0.36015$.

$$\approx 0.360$$

Exercise 20 - Two variables

Terms: $(2x)^4$; $4(2x)^3(-3y)$; $6(2x)^2(-3y)^2$; $4(2x)(-3y)^3$; $(-3y)^4$.

$$16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$