

Precalculus - Complex Numbers and Complex Zeros - Lesson

Common Core N-CN.A.1, N-CN.A.2, N-CN.A.3, N-CN.C.7, N-CN.C.8, N-CN.C.9 · maths-pdf.fr/en/

A complex number $a + bi$ uses the imaginary unit i , where i squared equals -1 . Add and subtract the real and imaginary parts separately, multiply like binomials replacing i^2 with -1 , and divide by multiplying by the conjugate. A negative discriminant gives a conjugate pair of complex roots, and a degree- n polynomial has exactly n complex zeros.

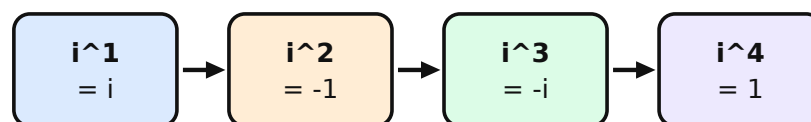
Precalculus · Common Core State Standards: **N-CN.A.1, N-CN.A.2, N-CN.A.3, N-CN.C.7, N-CN.C.8, N-CN.C.9**

Key facts

- $i^2 = -1$, and powers of i repeat in a cycle of 4: $i, -1, -i, 1$.
- The conjugate of $a + bi$ is $a - bi$, and $(a + bi)(a - bi) = a^2 + b^2$.
- Modulus: $|a + bi| = \sqrt{a^2 + b^2}$, the distance from the origin in the complex plane.
- A negative discriminant gives two complex roots that are conjugates.
- Fundamental Theorem of Algebra: a degree- n polynomial has n complex zeros counting multiplicity.

What is the imaginary unit?

No real number squares to -1 , so mathematicians define the imaginary unit i by $i^2 = -1$, or $i = \sqrt{-1}$. A **complex number** has the form $a + bi$, where a is the real part and b is the imaginary part. For a negative radicand, $\sqrt{-n} = i\sqrt{n}$, so $\sqrt{-25} = 5i$. The powers of i repeat in a cycle of four, so to find i^n divide n by 4 and use the remainder.



Because the cycle has length 4, $i^{27} = i^3 = -i$ (27 leaves remainder 3).

How do you add, multiply and divide complex numbers?

Add and subtract real and imaginary parts separately: $(2+3i)+(1-2i)=3+i$. Multiply like

binomials and replace i^2 by -1 : $(2+3i)(4-i)=8-2i+12i-3i^2=11+10i$. The **conjugate** of $a+bi$ is $a-bi$, and the product of a number and its conjugate is the real number a^2+b^2 .

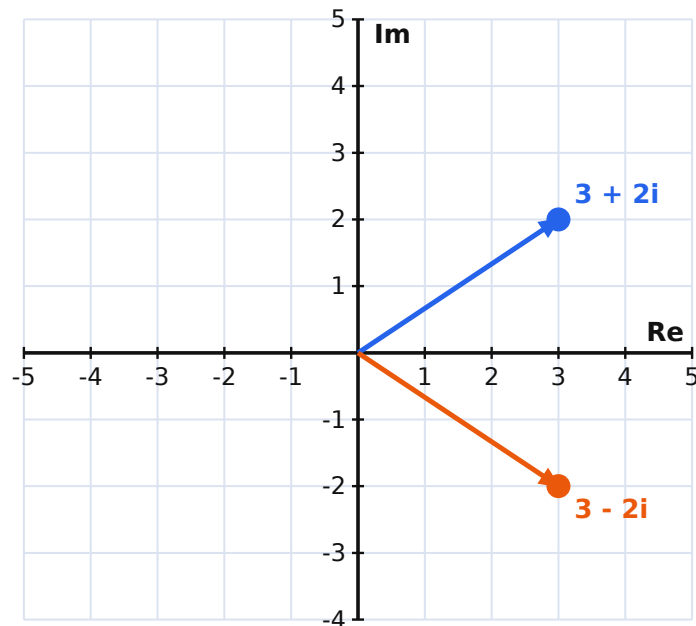
Dividing complex numbers

Multiply the numerator and denominator by the conjugate of the denominator:

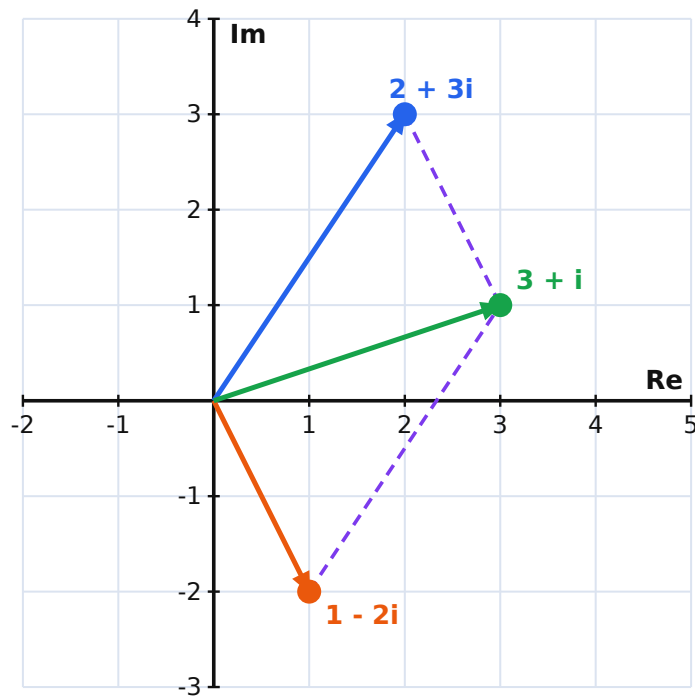
$$\frac{3+2i}{1-i} = \frac{(3+2i)(1+i)}{(1-i)(1+i)} = \frac{1+5i}{2}.$$

How do you picture a complex number?

In the complex plane the horizontal axis is real and the vertical axis is imaginary, so $a+bi$ is the point (a,b) . The **modulus** is the distance from the origin, $|a+bi| = \sqrt{a^2+b^2}$. A number and its conjugate are mirror images across the real axis.



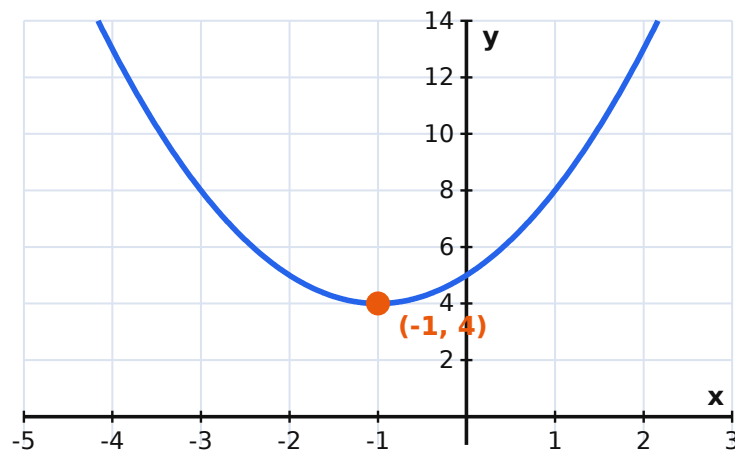
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Adding complex numbers is vector addition.

How do complex numbers solve quadratics?

If the discriminant $b^2 - 4ac$ is negative, a quadratic has two complex roots, always a conjugate pair when the coefficients are real. For $x^2 + 2x + 5 = 0$, the formula gives $x = \frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2i$. The graph never meets the x -axis.



$y = x^2 + 2x + 5$ has no real zeros; its complex zeros are $-1 \pm 2i$.

Complex numbers are not just a classroom idea. Engineers use them to describe alternating current, where a resistor adds a real part to the impedance and a coil or capacitor adds an imaginary part. Adding impedances in series is exactly the addition you practiced above, and the modulus gives the overall size of the opposition to current. Whenever a quantity has both

a size and a direction or phase, a complex number is a compact way to record it.

The Fundamental Theorem of Algebra states that every polynomial of degree $n \geq 1$ has exactly n complex zeros counting multiplicity. For real coefficients, non-real zeros come in conjugate pairs.

Questions about complex numbers and complex zeros

What is i squared?

By definition i squared equals -1 . This makes it possible to take square roots of negative numbers, such as $\sqrt{-9} = 3i$.

How do you find a high power of i?

Divide the exponent by 4 and use the remainder: remainder 0 gives 1, remainder 1 gives i , remainder 2 gives -1 , and remainder 3 gives $-i$.

Why multiply by the conjugate when dividing?

The product of a complex number and its conjugate is the real number $a^2 + b^2$, so the denominator becomes real and the quotient can be written as $a + bi$.

What is the modulus of a complex number?

It is the distance from the origin to the point in the complex plane. For $3 + 4i$ it equals $\sqrt{9 + 16} = 5$.

Do complex roots always come in pairs?

For a polynomial with real coefficients, yes. If $a + bi$ is a zero with b not zero, then $a - bi$ is also a zero.